Rethinking the Arc: An Econometric and Game-Theoretical Analysis of the NBA's Three-Point Reliance and Proposed Rule Changes

BY

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Pledge:

This paper represents my own work in accordance with university regulations.

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Artificial Intelligence Disclosure: I disclose that I used Generative AI in the writing of this paper under the guidance of, and with the full knowledge of my faculty advisor. I assert that my use of Generative AI has been documented and detailed within this paper and that all prompts have been saved and can be produced on demand, if required.

Justin Milligan

Abstract

Over the past decade, the National Basketball Association (NBA) has witnessed a historic rise in three-point shooting frequency, raising concern over the direction of the game. Fueled by a more comprehensive understanding of basketball analytics, this trend has led to the decline of the mid-range game and has produced more repetitive shot selections. Currently, over 42% of total shot attempts are from behind the three-point line, and the majority of the remainder are taken in the paint. My thesis investigates the degree to which three-point shooting is impacting the game today, and proposes two rule changes to mitigate this impact: the extension of the three-point line and the addition of a four-point line. To do this, I use a combination of econometric, statistical, and game-theoretical methods to motivate, establish, and analyze what these rule changes would look like over time. The results suggest that shifting the three-point line back to 26 feet and eliminating the corner three would return shot distributions to a 2011-2012 level, the season where ratings peaked since the turn of the century. Additionally, this paper analyzes the dynamics of a four-point line and its potential ramifications. After taking into account how player skill would develop over time, the results suggest placing a four-point line at 33 feet. While both of these lines would significantly decrease three-point shooting frequency, I argue that extending the three-point line to 26 feet most directly addresses the league's current issues. Overall, this thesis explores how economic strategies can depict the three-point reliance in the NBA, and analyzes the potential impact of rule changes.

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1 Introduction

As the average viewer's attention span continues to decrease, professional sports are rapidly evolving. Baseball has implemented the pitch clock, football has introduced expedited review, and hockey has embraced a three-on-three overtime format all in an effort to keep fans engaged. Basketball is certainly not exempt from this. Over the past few years, the NBA has introduced several rule changes such as transition take foul rule and the fourteen second shot clock reset. Despite these efforts, NBA regular season viewership has declined since the 2019-2020 season (Statista Research Department, 2024). According to a fan study conducted in 2021, 67% of respondents agreed that NBA games are less exciting than they were ten years ago (Bonnstetter, 2021). The league now finds itself at a pivotal moment in time; player skill is at an all time high, yet ratings and fan entertainment are not following the same trajectory. Many fans believe the bland style of play is to blame; successful teams have shifted toward prioritizing exclusively three-point shots and high-percentage twos. This has led to a historic increase in the amount of three-point shots taken, and a visible decrease in offensive creativity throughout the NBA, specifically in the mid-range. The league may shortly need to implement a rule change or risk a severe decline in fan interest. Two main proposals have emerged to solve this issue: one suggests rewarding elite shot-making ability through the addition of a four-point line, and the other proposes pushing the current three-point line backward to discourage high frequency three-point shooting. Through this research, I provide economic insight to help evaluate these proposals. In particular, this paper uses econometric and statistical methods to assess how each proposal might be implemented, gauge their feasibility, and project their broader ramifications on the game of basketball.

In order to achieve this goal, I first analyze the extent to which the present three-point shot is valued by both players and teams. Included in these analyses are how shooting threes affects point differential, player salary, and team success. These regressions and statistical analyses provide clarity on the current state of the NBA, illustrating the strong incentives associated with shooting more three-pointers. Then, I use historical data and statistical methods to model the future of the league, conditional on implementing the four-point line or the three-point line distance change. By modeling these hypothetical futures, I am able to discover optimal solutions for certain crucial aspects of the proposals, namely the optimal distance for these lines to be defined. With this information, I then analyze the game theoretical applications of these newfound solutions. The model I use is centered around a constraint optimization problem, utilizing past data to understand the efficiencies and inefficiencies of teams in certain parts of the court. I first model the current NBA, then switch the parameters to account for the hypothetical rule changes and compare the results. This part of the paper is pertinent to developing an opinion on whether or not these changes would be beneficial to the league. Throughout the paper, I discuss both qualitatively and quantitatively why this drastic increase in three-point shooting may be a net negative for the NBA, and how the rule changes can help fix this problem. Together, this research and discussion will provide an economic spin on understanding the evolution of the NBA and what potential rule changes could mean for the future.

1.1 Institutional Background

From the inception of the NBA in 1946, the game has undergone a series of transformations, both stylistically and structurally. Perhaps the largest of these transformations took place at the beginning of the 1979-1980 season with the introduction of the three-point line. The line is currently positioned as an arch, 22 feet away from the basket in the corners, and 23 feet 9 inches away at the top, resulting in a court with the following dimensions:

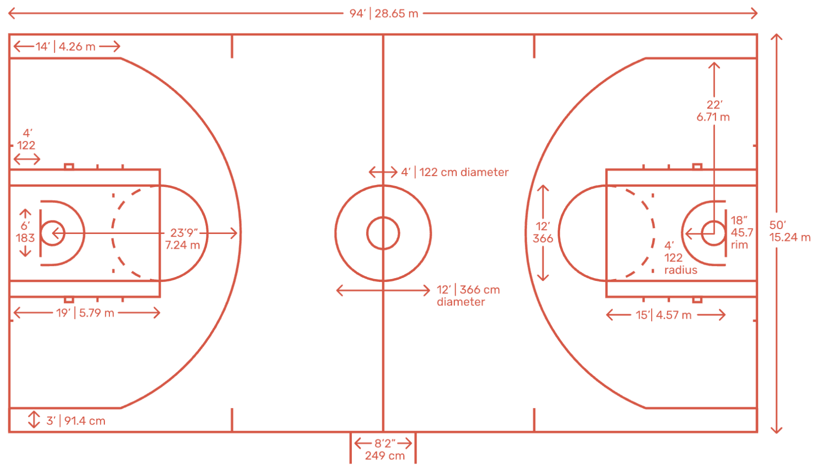

Figure 1: NBA Court (Current Dimensions)

Implementing this line was an attempt to increase excitement and reward the shot-making ability of skilled players. Originally, this change did not have much impact on the game; it took almost twenty years for the league to average just ten three-point attempts per game (StatMuse, 2025). Since then, however, the NBA has reaped the dividends of talent like Stephen Curry taking the league by storm in the 2010s through his supreme three-point shooting skill, despite lacking in height and athleticism. Curry quickly became the most popular jersey sale in the NBA after his unbelievable back-to-back MVP seasons, inspiring those around the world who may have previously thought their height was a barrier to entry into professional basketball (Barrigon, 2023). Although the league was already following a positive trend in three-point shooting, his success was undeniably a catalyst for the transformation of the professional game. This has led to a generation of players who shoot more threes than ever. However, the success of high-volume three-point shooting players/teams coupled with an increase in analytical study of basketball has led to a potential problem in the league.

1.2 The Problem

Over time, the three-point shot has become increasingly prevalent, with the 2024-2025 season seeing teams shooting anywhere from 30-50 three-point shots a game. For reference, in 1979-1980, the league-leading Los Angeles Clippers shot 6.1 three pointers per game. (StatMuse, 2025). This range equates to approximately 30-60% of teams' shot attempts coming from beyond the three-point arch, depending on the team. Using team-by-team data from 2012-2022, I created the following graph to illustrate the league-wide trend over time:

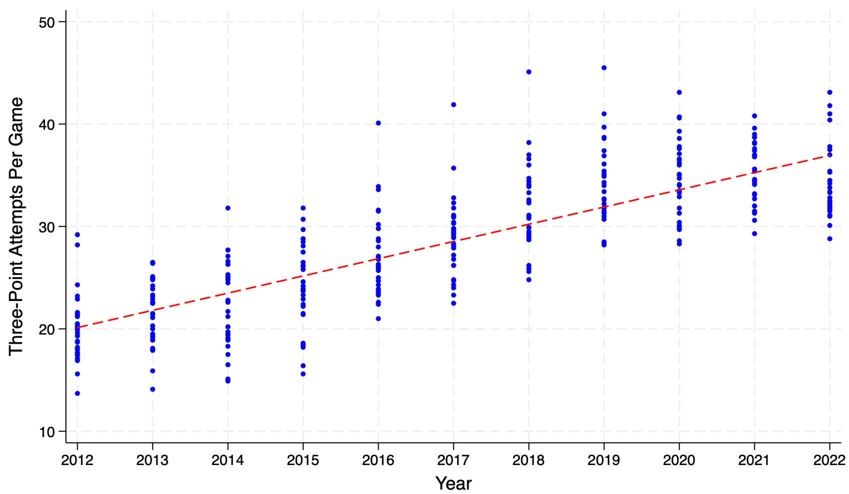

Figure 2: Scatter Plot of Three-Point Shot Attempts Per Game (Team Observations)

In just this decade, a league-average team went from shooting 20.0 three-point shots per game to 34.2, a 71% increase. This number has since increased to 37.5 per game in 2024-25 at the time of this paper being written (StatMuse, 2025). On the surface, there is nothing inherently wrong with teams shooting more threes. This is a strategy, and if teams consider it valuable, it is certainly within their jurisdiction to continue this trend. However, this stunning increase has become part of the national conversation when discussing potential reasons for ratings falling. While watching Steph Curry hit unbelievable three-point shots is entertaining, watching every player on the court shoot threes like they're Steph Curry might not be. The lack of differentiation in shot selection has been amplified, as more players have been placed in the "catch-and-shot" role, once reserved for only the best shooters. The Bonnstetter survey exemplifies this narrative; the following figure displays the justification provided by the respondents who believed that the game is less entertaining now than it was ten years ago:

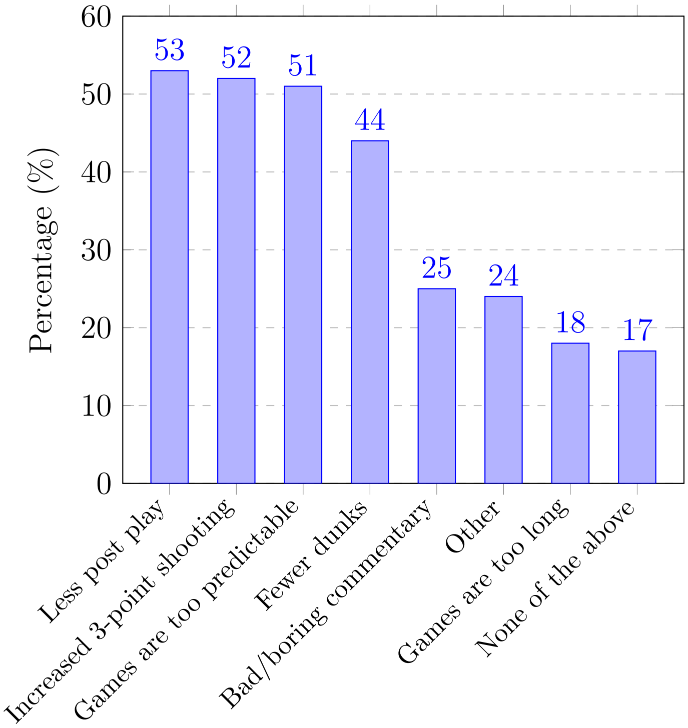

Figure 3: Bonnstetter Survey Results (What Factors Contribute to This Belief?)

Most directly, a staggering 52% of these fans believe the increase in three-point shooting is a contributing factor to their negatively trending opinion of the NBA. Additionally, the decline in post play (noted as a contributing factor for 53% of fans) is directly tied to the rise in analytical understanding of basketball I will later discuss; players simply aren't efficient enough from the mid-range to justify high quantities of shots to come from this region.

On top of these more qualitative lines of thinking, I later (Subsection 4.1) analyze how three-point shooting necessarily implies increased variance in point differential, leading to a lower number of close games. The results suggest that not only is the variance of point differential higher with increased three-point shooting, but the absolute value is as well. It is safe to assume that the audience typically enjoys a closer game, thus implying that this ramification of three-point shooting has a negative impact on fan experience. Further, in Subsection 4.2, I show that both players (via salary) and teams (via wins) are incentivized to continue shooting more threes. This, alongside the game-theoretical incentives I examine in Section 5 and Section 6, combine to explain why I consider the three-point shooting rate increase to be a problem in the current NBA.

1.3 The Solution(s)?

For the last decade, fans and media members have discussed potential rule changes surrounding the three-point shot, understanding that this continuously increasing trend is likely reflective of an imbalance in the game of basketball. As previously discussed, the viewership experience of basketball is more entertaining with shot creativity and variability; shooting the exact same looking shot from the three-point line is not aesthetically enjoyable. With this being said, the two most prominent rule change suggestions are adding a four-point line, or moving the three-point line backward. Importantly, these are not the only two potential solutions, but they are the two that I focus on in this paper.

1.3.1 The Four-Point Shot

Teams have continued improving at creating extremely efficient shots, and oftentimes these look very similar throughout the game (threes, dunks, layups). With this in mind, the four-point line would be a great way to add variation in shot selection, without penalizing players for getting better at shooting threes. Some positive externalities include increased spacing and more intriguing late game situations, but alongside these is a further increased variance of point differential, and even more distance shooting.

In the 2024 season, the Philippine Basketball Association (PBA) implemented the four-point line. This was the first professional league to do so in competitive game play. We have also seen versions of it by the Harlem Globetrotters, the Big 3, and in the Women's National Basketball Association (WNBA) All-Star Game. The PBA placed the four-point line at a length of 27 feet, and saw a league average of 23.9% on 5.8 attempts per game (Caramoan, 2024). Good or bad, the introduction of the four-point line created a dynamic different from anything seen in a basketball game. As coach Charles Tiu states, "a four-point lead is no longer safe at the end of the game because it's just a one possession game" (Caramoan, 2024). In general, the four-point line is an intriguing way to lean into the newfound deep shooting skill of today's players. Later in the paper I take a closer look at what exactly the addition of a four-point line would look like in the NBA.

1.3.2 Changing the Distance of the Three-Point Line

In a different light, some believe that long jump shots need to be limited because they are significantly less entertaining than other forms of scoring (dunks, layups, post-scoring), and create less competitive play (higher variance of point differential). The league has reached the point where most players, if given any space at all, have permission to shoot a three-pointer. Thus, increasing the requisite skill to shoot these shots by moving the line farther away is a potential solution to this issue. In the Bonnstetter survey, 62% of respondents agreed that the NBA should move the line back, with another 10% being neutral on the issue. This is a reflection on both the idea that many fans tend to not enjoy the current volume of distance shooting, and the overwhelming likelihood that this change would have an immediate and tangible impact on the problem.

One potential objection to this type of rule change is the impact on the historical continuity of statistics, and what it means for comparing players of different eras. However, there is precedent for changing the line's distance. In the early 1990s, the NBA believed that lack of scoring was an issue for the future of the league. At this time, teams were shooting less than 10 three-point shots a game at around 33% efficiency (StatMuse, 2025). To combat this, the three-point line was moved forward to 22 feet for three seasons. This increased both quantity and efficiency of three-point shots being taken, yielding an average of 16 attempts per game at 36%. The existence of these three years proves that is certainly within the scope of the league's authority to move the line, although a backwards move has not yet occurred. As the paper progresses, I will discuss more specifically what this would entail and how it would impact the future of the league.

2 Literature Review

This section reviews the relevant material surrounding the NBA in economic literature. Several papers have applied economic theory to basketball, offering further insight into strategies which maximize shot efficiency, personnel decisions, and win probability. As sports analytics continue to advance in accuracy and applicability, this paper aims to contribute a unique perspective by applying existing theory and data to address hypothetical rule changes such as the four-point line and the extended three-point line. The following existing literature provides a foundational understanding of the underlying dynamics of the current state of basketball, and my goal is to build on this for the remainder of my paper.

Extensive research on game theory applications to basketball have been and continue to be conducted. From this research, teams have evolved their strategies over time, striving closer to optimality. This idea of optimality can be applied to both general shot selection and late-game decision-making. Franklin Kenter's paper titled "An Analysis of the Basketball Endgame: When to Foul When Trailing and Leading", models end-of-game basketball as a combinatorial game (Kenter, 2015). He then graphs the optimal shooting/fouling decision conditional on point differential and time remaining. This model is then compared with historical data, showing that it is an accurate representation of the optimal strategy in practice. That is, teams whose decisions follow Kenter's suggestions have a higher win probability in the aggregate. Another great paper titled "Evaluating NBA end-of-game decision-making" written by Patrick McFarlane supports idea more directly (McFarlane, 2019). This paper details the development of an "end of game tactics metric (ETM)" allowing the evaluation of teams' decision-making to be analyzed more accurately. Again, the results suggest that following these more analytical approaches to basketball (as I will later depict in a more broad manner) yields a higher success rate. Clearly, the suggestions provided by these two models would be vastly different if a four-point line were introduced; situations such as being down four (described earlier by Charles Tiu) become extraordinarily winnable, and the conditions for fouling would change significantly (McFarlane, 2019). More subtly, pushing back the distance of the current three-point line would also have an impact on these decisions, as adjusting these models to a less efficient shot would create new and intriguing insights. Ultimately, the rule changes my paper proposes would necessitate a reevaluation of late-game strategy.

As previously mentioned, economic theory has not only been used to analyze end of game situations, it has also provided great insight for general game strategy. This includes but is not limited to shot selection and personnel decisions. Kevin Lu's paper titled "The Valuation of Shots in the NBA" provides an extremely clear account of the relationship between "optimal" shot selection and winning (Lu, 2024). He credits Daryl Morey, former general manager of the Houston Rockets, for leading the analytics-driven approach to shot selection in the NBA. Under Morey, the Rockets saw huge successes and were taking extraordinarily efficient shots in comparison to their competitors, including many corner three-pointers and layups. For the purposes of my paper, the important takeaway is the efficiency of such shots; the NBA with a three-point line has been "figured out" to an extent. These short three-pointers and layups are significantly more efficiency than any other shots on average. Since this Rockets team, more teams are attempting to create a more efficient distribution of shot selection. This ties back to the Bonnstetter survey, in Figure 3 we see that many fans believe the post game (mid-range) has been underutilized in the current NBA; this is a direct result of the current analytical movement.

2.1 Fichman & O'Brien Paper

In the game theoretical section of this paper, Fichman and O'Brien's paper titled "Optimal Shot Selection Strategies for the NBA" is referenced prominently. This paper calibrated a mixed strategy equilibrium of shot selection using data from the 2015-2016 NBA season (Fichman and O'Brien, 2019). To do so, they categorized every shot into a different "zone", defined by finite regions on the court. Once the shots were placed into one of the 11 zones, they then collected data on each zone, seeking to discover how often and how efficiently shots are taken from them. This eventually led to a constraint optimization problem where they maximize the difference of the offensive and defensive payoff sharpe ratios to understand the "optimal" weighting of shots per zone. The results are perhaps not shocking; they suggested that in the 2015-2016 season, teams took a significantly lower than optimal number of three-point shots. In fact, they state "the results suggest that future NBA 3-point averages are headed higher to 37.9%", from only 28.6% of shots being three pointers in the 2015-2016 season. In less than a decade, the 2023-2024 season has exemplified (and even outdone) their prediction; 39.4% of shots taken were three-pointers. At the time of this paper being written, over 42% of shots taken in the 2024-2025 season are three-pointers. Although these results are shocking and interesting, the methodology of Fichman and O'Brien's work is what will most impact my paper. I will use the strategy they depicted to create an optimal shot distribution between the zones, depicting how this has changed over time, then apply hypotheticals (such as a four-point line and an extended three-point line) to understand how these rule changes would impact the game. I will further explain my methodology with regard to this paper in Section 6.

Collectively, these papers exemplify the fact that the NBA has been trending toward optimality for a while now; teams have a solid understanding of strategy to maximize efficiency and win probability. My proposed rule changes would certainly shake all of this up, and the dynamics of basketball would need to be re-understood. End of game situations would need to be re-imagined, and data on success rates for certain shots/strategies would need to be collected to accurately predict these dynamics. The four-point line would likely increase on-court spacing and alter the aesthetics of offensive gameplay. A longer three-point line would have teams reconsidering who and how often they truly should shoot three-pointers. Defensive strategy would vary significantly, and offensive structure would certainly rely less on running around the three-point arch. Although the introduction of the four-point line or moving the three-point line back would create many uncertainties, this existing literature provides a foundational understanding of basketball, and can certainly be adapted and applied to account for future rule changes.

3 Data

This type of research necessitates the use of a significant amount of data. This ranges from league-wide statistics such as team efficiency metrics, win totals, and salary information, to a more narrow scope in shot-level data. While the more generalized information will garner an understanding of how the league currently operates, I use the shot-level data to delve deeper into specific shot locations and how these league-wide statistics can vary internally. In addition, I also utilize Google Trends data to provide a proxy for fan interest. As a whole, this section will elaborate upon how I collected and utilized each subset of data.

3.1 Basketball Reference

The league-wide data and statistics referenced in this paper are predominantly drawn from Basketball Reference, an extraordinarily popular website for all things basketball. In their publicly available database, Basketball Reference provides player stats, team stats, records, individual game information, and much more. I draw heavily from specific sections of this website to develop several custom datasets which will be useful to analyze different aspects of this paper.

3.2 Historical Efficiency Data

In order to understand how the introduction of the three-point line impacted player efficiency, I needed league-level data on two-point shooting prior to 1979. Basketball Reference provides this, with data from the inception of the NBA which I was able to download and transform into a usable dataset for the purposes of my paper ("NBA & ABA League Index", 2025). The dataset I created contains year-over-year information on major statistical categories including field goal attempts, three point attempts, field goal makes, three point makes, and several other useful variables from both before and after 1979. An important consideration is the dynamic between adding an additional line and the percentage of existing shots, i.e. what happened to two-point efficiency when the three-point line was added? I created the following graph with this data, the red line marking 1979, the year of the inception of the three point line:

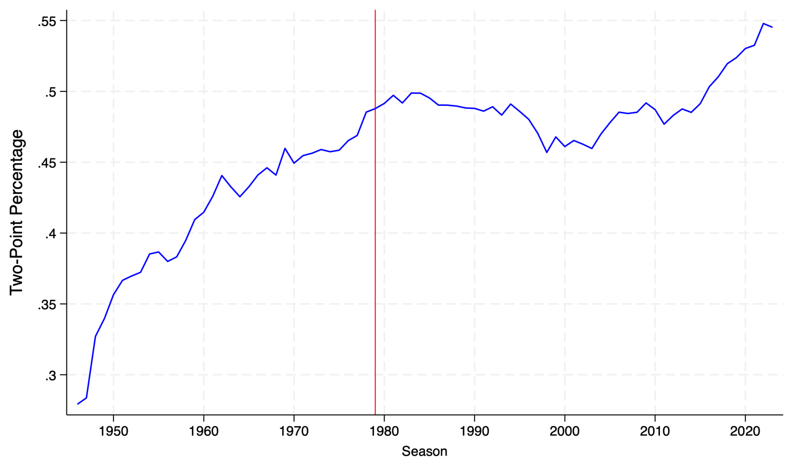

Figure 4: Line Graph of Historical NBA Two-Point Percentage over Time

One thing is immediately recognizable on this graph: there is a positive relationship between time and two-point efficiency. This makes perfect sense, players have become more skilled shot-makers over time due to more efficient training, increase in playing population, and breakthroughs in sports science, among other factors. It is not directly clear what influence the addition of the three-point line had, but later in the paper I dive into this further. Having this data, however, is key to modeling the future. In addition to this, I was also interested in how the three-point shot developed after its inception. With the data available, I was able to create two graphs; one depicting three-point efficiency over time, and the other showing three-point attempts over time.

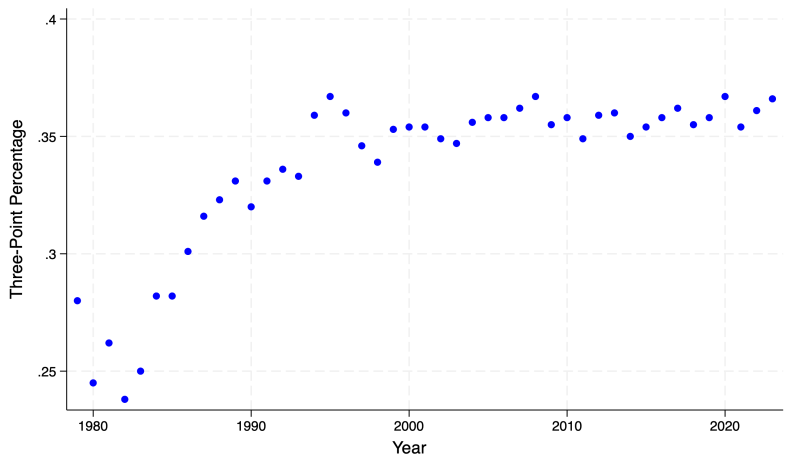

Figure 5: Scatter Plot of Historical Three-Point Percentage (League-Wide) over Time

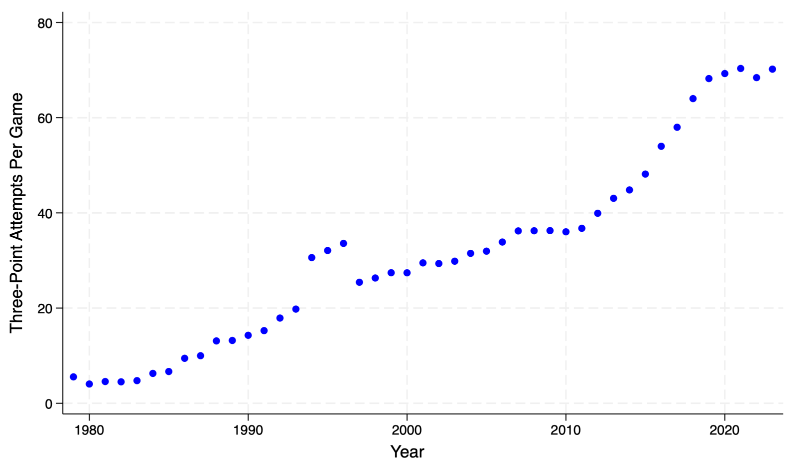

Figure 6: Scatter Plot of Historical Three-Point Attempts Per Game(Team Observations)

These two graphs show two important motivators for this research; three-point percentage seems to have plateaued at some type of equilibrium, yet the attempts per game continue to increase. This relationship is impactful because it supports the idea that even if the ability of the players to make threes remains relatively constant in practice, it has been proven to be optimal to continue shooting more. This indicates that an imbalance in the game is slowly being realized through a more analytical approach to game strategy.

3.3 Google Trends Data

In order to further understand the dynamics currently influencing viewer interest in the NBA, I manually collected data from Google Trends on each of the 30 NBA Teams. Google Trends is a free website which analyzes the popularity of search queries on Google. To obtain this data, I inputted each of the NBA Teams' names, and downloaded the data for the dates corresponding to approximately the first quarter of the 2024 NBA season (by date, but not necessarily by games played for each team). I merged this with another dataset which includes all games played this year so far, separating each game by team. The game data was sourced from Basketball Reference ("NBA & ABA League Index", 2025). A summary of these statistics is provided below:

Table 1: Summary Statistics for Google Trends Dataset (2024 Game Data Included)

Variable	Obs	Mean	Std. Dev.	Min	Max
Points	780	112.9	12.6	80	145
Attendance	780	17933	1544	13113	22062
Three-Point Attempts	780	75.1	9.3	51	108
Overtime (Binary)	780	0.05	0.2	0	1
In-Season Tournament (Binary)	780	0.17	0.38	0	1
Game Time (24h clock)	780	19.8	1.6	12	22.5
Trends	780	44.9	20.0	0	100
Back-to-Back (Binary)	780	0.18	0.38	0	1
Win (Binary)	780	0.5	0.5	0	1

Later in this paper I analyze these relationships, specifically how each category influences the Trends variable. Trends is used as a proxy for fan interest at that given time, as ratings on specific games are unavailable. These results will be displayed in Section 4, which aims to motivate this research.

3.4 Historical Wins, Shooting, & Salary Data

In order to conduct this research, I downloaded data on thousands of games played on Basketball Reference. With these data, alongside various other statistics available on their website, I created several small datasets. First, I created a dataset which merges team points per game and team three-point attempts per game. In Subsection 4.1, I use eight years of this data in order to understand how point differential and three-point shooting are related.

In addition to this, I sought to understand the relationship between wins, salary cap spending, and three-point attempts. To do so, I manually created my own datasets using TeamRankings, Spotrac's Salary Cap Tracker, and Basketball Reference's NBA League Index ("NBA & ABA League Index", 2025; Spotrac, 2025; TeamRankings, 2025). The first of these two datasets includes six variables: Wins, ThreePA, TotalCap, Year, diffThreePA, and diffTotalCap. TotalCap represents the total amount a team allocates toward player salaries in a given year, and ThreePA represents the average three-point attempts per game for a given team. The "diff" variables compare a given team's value to the league average for that year, shown below:

$$\begin{aligned} \text{diffThreePA}_{i,t} &= \text{ThreePA}_{i,t} \ - \ \overline{\text{ThreePA}}_t, \\ \text{diffTotalCap}_{i,t} &= \text{TotalCap}_{i,t} \ - \ \overline{\text{TotalCap}}_t, \end{aligned} \tag{1}$$

With these definitions in mind, we should expect the "diff" variables to have means of approximately zero. Below are the summary statistics for this dataset:

Table 2: Summary Statistics for Historical Wins & Salary Cap Dataset

Variable	**Obs**	**Mean**	**Std. Dev.**	**Min**	**Max**
Wins	330	40.518	12.013	15	73
ThreePA	330	28.550	6.682	13.7	45.5
TotalCap	330	98.384	31.236	36.6523	184.0248
Year	330	2017	3.167	2012	2022
diffThreePA	330	0.013	3.736	-8.5742	13.1936
diffTotalCap	330	0.000	12.9229	-49.7088	43.8028

One important note is that TotalCap and diffTotalCap are depicted in millions of dollars. These are later used to control for spending in regressions, as these values together should fairly accurately encapsulate the financial situation of a given team. While diffThreePA is not exactly zero, the difference is negligible, and likely due to rounding in the dataset. As the paper progresses, I will analyze the relationship each of these variables have on Wins, specifically trying to understand how the amount of threes a team is shooting influences team success.

The next small dataset I created includes inflation-adjusted salary and three-point attempt rate for each player. The following is a summary of the important data:

Table 3: Summary Statistics for Attempt Rate / Inflation Adjusted Salary Dataset

Variable	Obs	Mean	Std. Dev.	Min	Max
Inflation Adjusted Salary (Millions)	3623	7.7	9.2	0.04	51.4
Three-Point Attempt-Rate	3623	0.34	0.21	0	1
Games Played	3623	53.9	22.2	2	83
Age	3623	26.1	4.1	19	43
Year	3623	2018.1	2.6	2014	2022

In subsequent sections, this data will be used to analyze the relationship between inflation adjusted salary and three-point attempt rate over time. As the value of threes increase over time, I would predict that having star players who attempt a lot of threes will become more and more valuable. In section Subsection 4.2, I analyze this relationship, and discuss the ramifications of this information in the scope of my paper.

3.5 Shot Data

Next, I obtained two datasets each including play-by-play shot observations from different time periods. The first of the two was scraped from Basketball Reference by user Bhavya on Kaggle (Bhavya, 2022). This dataset includes every shot from every NBA game from 2000-2022, who took it, the result of the shot, and the distance. Understanding the changes in this data over time will be pivotal to comprehending the development of the NBA.

In addition to this dataset, I purchased 2023-2024 play-by-play data from BigDataBall (BigDataBall, 2024). In this data, I have access to shot type, x and y coordinates of where the shot was taken, distance, player, time in the game, score, and more important variables.

This will give me a great foundation for my analysis moving forward. With this shot data, I created two intriguing figures, the first of which depicts the efficiency of three-point shooters conditional on distance for the 2023-24 season, shown below:

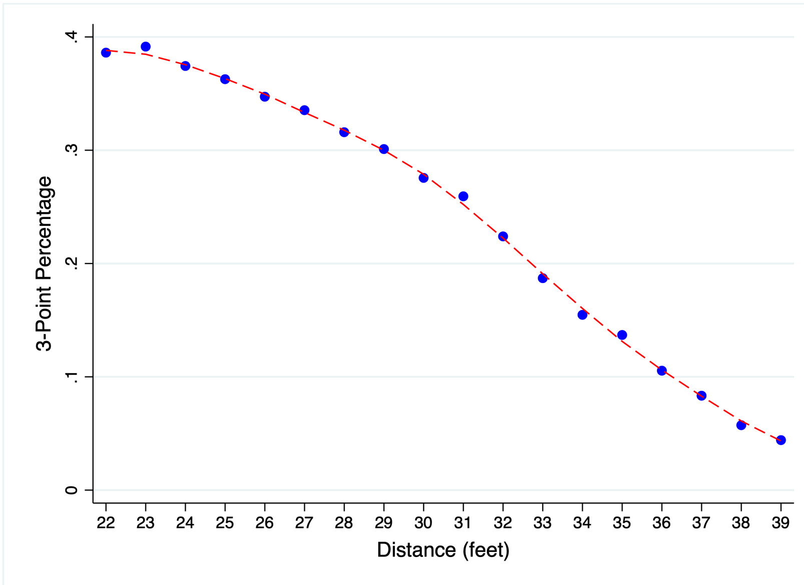

Figure 7: Scatter Plot of 2023-24 Three-Point Percentage by Distance

I refer back to this figure throughout the paper, as it provides a measure of the skill level of NBA players currently. From this, I can extrapolate predictions for future hypothetical scenarios. Notably, when shooting from directly behind the three-point line (from 22-24 feet), players are extremely efficient, with their percentages reaching nearly 40% on average. The efficiency then steadily decreases over distance, with players shooting from over 35 feet hitting under 10% of their shots. In Section 5, I apply this relationship to help gauge where the four-point line and the extended three-point line should be placed.

Furthermore, this data can provide insights on all shots, not just three-pointers. While the main focus on this paper is on three-point shooting, understanding how the dynamics and efficiencies of other zones on the court interact with and compare to distance shooting is pivotal for many parts of the paper, namely Section 6. With this same dataset, I created another plot which includes all shots taken during this season, color coded by make(green) or miss(red). The result is shown below:

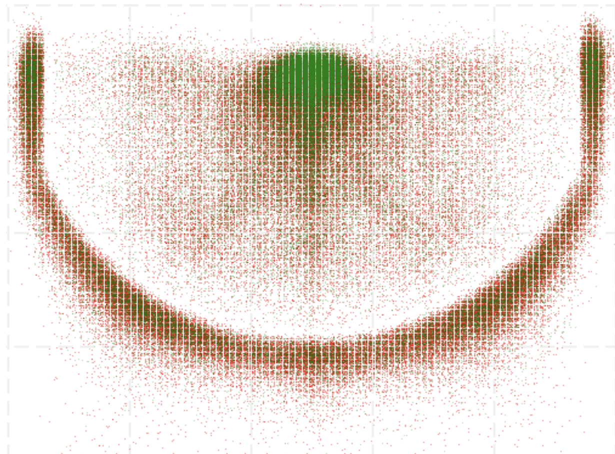

Figure 8: Scatter Plot of 2023-24 Shots

This figure exemplifies many key ideas I presented in the introductory stages of this paper. Rewinding to my claims in Section 2, this shot chart displays the efficiency of corner three-pointers and layups/dunks, while showing how almost all mid-range shots are quite inefficient. Additionally, it visualizes the relationship in Figure 7, with the more efficient three-point shots coming directly behind the line (and thus further outwards is more red / less efficient). More generally, the density of shots concentrated around the three-point line and at the basket is jarring; especially when comparing it to the exact same plot using data from the 2000-2001 season:

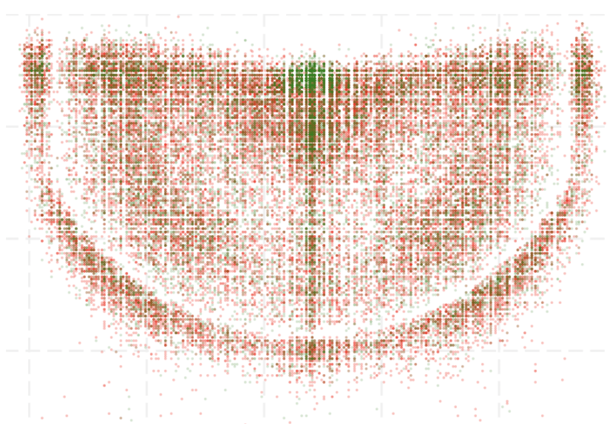

Figure 9: Scatter Plot of 2000-2001 Shots

Comparing these two shot charts, the decline in the mid-range and advent of three-point shooting are simultaneously visible. Alongside these two takeaways, shots around the rim seem to have improved in efficiency and frequency over time. This is likely a result of the increased spacing generated by three-point shooting, and overall skill increase. These datasets will be crucial later in the paper, and simply plotting them visualizes a notable decrease in shot diversity.

In order to use this data for the game theoretical aspect of this paper, I needed to add a few important categorizations and calculations. Using the x and y values given in the datasets, each shot was given a "zone", ranging from 1-11, uniquely identifying areas of the court. The following figure depicts where the zones were defined:

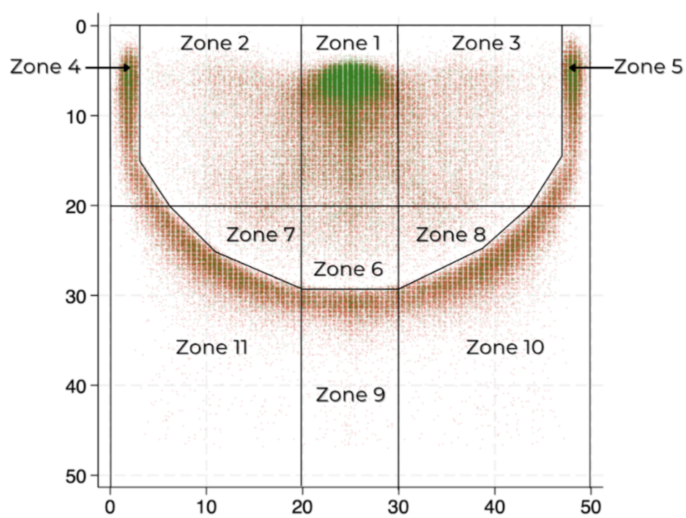

Figure 10: 2023-2024 Shot Chart With Zone Definitions

These zones will be crucial in defining and understanding how teams could optimize their shot selection strategies under certain constraints and parameters. Once the zones were defined, I calculated each team's average points scored and shooting efficiency by zone, along with their standard deviations. It is important to note that zones 4, 5, 9, 10, and 11 are worth three-points per make. Thus, their typically lower efficiency should be considered in with this context. Below is a summary table of the results:

Table 4: Zone Summary Statistics

	Points	**Sd**	**Eff**	**Sd(Eff)**
Zone 1	46.545	9.742	0.589	.084
Zone 2	4.690	3.219	0.422	.246
Zone 3	4.346	3.109	0.418	.256
Zone 4	6.450	4.588	0.385	.241
Zone 5	6.756	4.626	0.380	.229
Zone 6	1.794	1.754	0.442	.402
Zone 7	1.208	1.356	0.391	.433
Zone 8	1.701	3.992	0.384	.437
Zone 9	4.372	3.690	0.345	.273
Zone 10	10.015	5.644	0.356	.171
Zone 11	10.762	6.009	0.355	.162

I will use this data as the foundation for the game theory analysis conducted in Section 6. Quantifying the efficiencies and inefficiencies in teams' shot selections is pivotal for the constraint optimization problem.

4 Motivation

This section will present statistical and econometric analyses which motivate the central topic of this paper; the extent to which three-point shooting impacts the NBA.

4.1 Increased Variance

An intriguing factor of the current NBA landscape is the increased variance implied by shooting more three-pointers. Assuming that the average fan enjoys watching a closer, more competitive basketball game, shooting more threes does negatively impact this aspect of the fan experience. The relationship between higher point differential variance and more three-point shooting can be proven statistically. To do so, I will quickly describe a hypothetical which illustrates this point. Two teams, A and B, score at the two extremes of three-point frequency. Team A shoots exclusively three-pointers (50 per game at 40%

efficiency), while Team B shoots exclusively two-pointers (50 per game at 60% efficiency). Note that their expected point totals are equivalent:

$$\mathbb{E}[\text{Points}_A] = 50 \times 0.40 \times 3 = 60$$

$$\mathbb{E}[\text{Points}_B] = 50 \times 0.60 \times 2 = 60$$

With this information, you can calculate the variance and standard deviation of the points scored for each team (treating points as a random variable).

$$\text{Var}(\text{Points}_A) = 3^2 \times \left[50\, p_A\,(1 - p_A)\right] = 9 \times 50 \times 0.40 \times 0.60 = 108$$

$$\text{Var}(\text{Points}_B) = 2^2 \times \left[50\, p_B\,(1 - p_B)\right] = 4 \times 50 \times 0.60 \times 0.40 = 48$$

$$\text{SD}(\text{Points}_A) = \sqrt{108} \approx 10.39, \quad \text{SD}(\text{Points}_B) = \sqrt{48} \approx 6.93$$

The important part here is that the variance and standard deviation of Team A's scoring are significantly greater than that of Team B. With this information in mind:

$$\begin{aligned}\text{Var}\big(\text{Point Diff}\big) &= \text{Var}(\text{Points}_A - \text{Points}_B) \\ &= \text{Var}(\text{Points}_A) + \text{Var}(\text{Points}_B) - 2\,\text{Cov}(\text{Points}_A, \text{Points}_B) \\ &\approx \text{Var}(\text{Points}_A) + \text{Var}(\text{Points}_B).\end{aligned}$$

This proof assumes that, in the aggregate, the covariance of teams' scoring is zero. That is, that how well one team is shooting does not influence the other team's shooting performance. Small variations from this assumption are certainly possible due to momentum, defensive prowess, and other factors, but over the entirety of the league it is likely a value equal to, or extremely close to, zero. Thus, the variance in point differential increases directly with the variance of each team's scoring. Since I established that variance of scoring increases with three-point frequency, you can conclude that three-point frequency increases variance in point differential.

4.1.1 Historical Analysis

To test this, I utilized Basketball Reference's historical NBA data. I identified two four-year spans with equivalent points per game(PPG), but significantly different three-point attempts per game (3PA). This quality allows me to control for the potential relationship between PPG and point differential. Below is a summary table from these seasons:

Group 1 (1983–84 to 1986–87)		
Season	**PPG**	**3PA**
1986–87	109.9	4.7
1985–86	110.2	3.3
1984–85	110.8	3.1
1983–84	110.1	2.4
Mean	110.25	3.4

Group 2 (2017–18 to 2020–21)		
Season	**PPG**	**3PA**
2020–21	112.1	34.6
2019–20	111.2	34.1
2018–19	111.8	32.0
2017–18	106.3	29.0
Mean	110.35	32.4

Table 5: Summary Table of PPG & 3PA by Group

Having this information, I can now analyze how point differential differs between the two groups. To do so, I created a binary variable (Group 2) which is equal to 1 if the game took place any time from the 2017-18 season to the 2020-21 season, and 0 otherwise. I subsequently ran the following simple regression:

$$\text{PointDifferential}_i = \beta_0 + \beta_1\,(\text{Group2}_i) + \varepsilon_i \qquad (2)$$

Table 6: Point Differential Regression Results

	Point Differential
Group 2	**0.957*****
	(0.166)
Constant	10.697***
	(0.116)
Observations	10,014
R-squared	0.0033

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Put simply, these results suggest that the mean point differential (in absolute value) of games in Group 2 was a statistically significant amount larger than the point differential of games in Group 1. Further, the coefficient of 0.957 represents about one point per game of increased mean point differential, or about an 8.95% increase from the mean of Group 1. The following table displays summary statistics for these point differentials (importantly including the standard deviation).

Group	**Mean**	**Std. Dev.**	**Frequency**
1	10.69691	7.937542	5,081
2	11.65376	8.639822	4,933
Total	11.16826	8.304306	10,014

Table 7: Point Differential Summary

To analyze the variance of these groups, I utilized Stata's "robvar" command. Using this command reports Levene's robust test statistic (W0) for the equality of variances between the two groups. Then, it reports the two statistics proposed by Brown and Forsythe that replace the mean in Levene's formula with alternative location estimators. The first alternative (W50) replaces the mean with the median, and the second replaces the mean with the 10% trimmed mean (W10) (StataCorp, 2013).

Test	Statistic	df	p-value
W0	26.12732	(1, 10,012)	0.00000033
W50	16.024108	(1, 10,012)	0.00006299
W10	22.728732	(1, 10,012)	0.00000189

Table 8: Robust Variance Test Results

The results of this variance comparison test suggest that it is very safe to assume the variance of point differential in Group 2 is greater than that of Group 1 (as suggested in the hypothetical). This, coupled with the fact that the mean point differential in Group 2 is higher, lends to games which are significantly less competitive (as determined by score) than before. While it is possible that other factors (i.e. competitiveness of players, officiating, etc) impact this as well, it is certainly true that three-pointers are having a significant negative impact on how close these games are on average.

4.2 Three-Point Valuation & Incentivization

With this in mind, I wanted to discuss and analyze how three-pointers are currently valued and incentivized. To begin on a team level, I looked first at how wins have been impacted by three-point attempts from 2013-2022. Using the data provided in Subsection 3.4, I ran a regression with the following functional form:

$$Wins_{it} = \beta_0 + \beta_1 \,(\text{diffThreePA})_{it} + \beta_2 \,(\text{diffTotalCap})_{it} + \sum_{y=2013}^{2022} \alpha_y \, Year_{y,it} + \epsilon_{it}. \quad (3)$$

Included in this regression are independent variables diffThreePA, and diffTotalCap, fixed effects for the year, and the dependent variable, Wins. The diffThreePA and diffTotalCap variables are used to compare to league average at the time, defined in Equation 1. Accounting for total cap spending will control for the (presumably positive) impact spending more has on winning. The year fixed effects control for any time-related variation. The regression results are shown below:

Table 9: Wins Regression (Independent Variable)

Variables	**Coefficient**	**Std. Error**
diffThreePA	**0.783*** **	(0.172)
diffTotalCap	0.113**	(0.050)
2020	-5.023*	(3.011)
Constant	40.905***	(2.129)
Observations	330	
R-squared	0.092	

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Note: Fixed effects for every year included (removed for succinctness)

Most importantly, there is a statistically significant positive relationship between the amount of three pointers a team attempts and the number of wins they end up with in a given season. That is, for every three-point attempt per game over league average, a team wins an additional 0.783 games. This is an extremely important result, supporting the previous claims made in this paper. Later, I will elaborate upon the positive relationship between ThreePA and Wins, as it has a fairly clear game-theoretical explanation. In addition to this, at the 95% confidence level, there is a statistically significant positive relationship between diffTotalCap and Wins. When a team spends $1 million over the league average cap spending, they win 0.113 more games on average. This makes sense, as more spending usually equates to better players. Lastly, I removed all of the year fixed effects from the table besides 2020, as it was the only year which had a statistically significant coefficient. This coefficient of -5 is perfectly rational; a normal NBA season has 82 games, but Covid-19 caused 2020 to only have 72 games, thus teams won five less games on average.

To build on this, I want to understand the relationship between player salary and three point attempt rate (the frequency at which a field goal taken is a three-pointer). Because I have proven that shooting a higher frequency of threes is valuable for a team, I expect to see that the three point attempt rate has a positive effect on salary, holding other variables equal. To ensure this was a representation of meaningful NBA players, I first dropped all observations who had played less than fifty games, and all observations with a player efficiency rating (PER) less than league average. Then, I created a discrete variable (one through five) defining each player by one position to account for any position biases with regard to salary. The following is the functional form of my regression model:

$$IASalary_{it} = \beta_0 + \beta_1(ThreePointAttemptRate)_{it} + \sum_{y=2012}^{2022}(\alpha_y \cdot Year_y) + \sum_{p=1}^{5}(\mu_p \cdot Pos_p) + \epsilon_{it} \quad (4)$$

I included fixed effects for the year again due to the possibility of this dynamic shifting over time, but this time also had fixed effects for the position as well. The results are shown below:

Table 10: Inflation Adjusted Salary Regression (Independent Variable)

VARIABLES	**Coefficient**	**Std. Error**
Three-Point Attempt Rate	**7.615*** **	(2.794)
2015	0.848	(1.552)
2016	2.472	(1.551)
2017	5.416***	(1.584)
2018	5.264***	(1.565)
2019	7.361***	(1.592)
2020	8.462***	(1.612)
2021	7.986***	(1.637)
2022	6.800***	(1.586)
Point Guard	7.587	(5.648)
Shooting Guard	5.829	(5.689)
Small Forward	11.55**	(5.718)
Power Forward	7.331	(5.666)
Center	4.527	(5.677)
Constant	0.996	(5.770)
Observations	917	
R-squared	0.126	

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

As expected, the relationship between attempt rate and salary is positive. This suggests that a player with a 100% three-point attempt rate would get paid $7.615 million more than a player with a 0% attempt rate, holding position and year constant. This is a quite strong relationship, indicating the significant value placed on three-point shooting by team management. The motivation behind both of these regressions is simple; I want to show that over the last decade, shooting more three-pointers has become increasingly valuable for both team success (wins) and player success (salary). Thus, these two entities will continue to follow these incentives until an upper bound is hit or the rules are changed.

Next, I looked at the Google Trends dataset from Subsection 3.3. Utilizing the Google Trends variable as a proxy, the main goal of this analysis is to see what underlying factors influence fan interest. Understanding how fans respond to changes in certain aspects of NBA games is an extremely valuable insight in this paper, especially if this shows a statistically significant relationship with regard to three-point shooting. To do so, I set up a fairly simple linear regression model, the equation and results shown below:

$$\begin{aligned} Trends = \beta_0 + \beta_1(Attempts) + \beta_2(Win) + \beta_3(Attendance) + \beta_4(OT) + \beta_5(IST) \\ + \beta_6(Time) + \beta_7(B2B) + \beta_8(GameNum) + \beta_9(Points) + \epsilon_{i,t}. \end{aligned} \quad (5)$$

Table 11: Trends Regression (Dependent Variable)

VARIABLES	**Coefficient**	**Std. Error**
3-point attempts	-0.0200	(0.0675)
Win	-4.338***	(1.209)
Attendance(1000s)	1.307***	(0.414)
Overtime	7.316***	(2.399)
In-Season Tournament	6.862***	(1.332)
Time	3.164***	(0.390)
Back-to-back	-2.674**	(1.322)
Game	-0.381***	(0.0653)
Points	0.0501	(0.0518)
Constant	-37.60***	(13.28)
Observations	780	
R-squared	0.595	

Note: Fixed Effects for all 30 teams included (removed for succinctness)
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

My original hypothesis would suggest the coefficient of Attempts should be negative. While it is negative, the relationship is not statistically significant. This is unfortunate, but could be for a plethora of reasons including an omitted variable or Trends simply not being a good proxy for fan interest. In general though, most of the other results make sense. Things that would drive fan interest; overtime games, In-Season Tournament games, and games played later in the night all have positive effects on the next day's Trends value. However, this regression suggests that wins surprisingly have a negative effect on fan interest the next day. Overall, more concrete ratings data is necessary to prove this hypothesis, but this Trends proxy has provided a promising initial result.

Through this section, I aimed to first prove that three-point shooting increases both the variance and absolute value of point differential. With this in mind, it should be alarming that team success is positively impacted by shooting more threes, as shown in Table 9. As if this wasn't enough, it is visible in Table 10 that players are incentivized to develop and shoot more threes, because three-point attempt rate has increasingly led to higher salaries. Combining these two, there exists negative result (increase in both variance and absolute value of point differential) which is incentivized on both a team and player level. This is concerning, and is why we have seen such a massive change in the NBA over the last decade. While the Google Trends regression unfortunately did not lend statistically significant results, this section as a whole provides justification for analyzing potential rule changes to limit three-point shooting frequency.

5 Analysis

This section provides necessary analysis which establishes a foundation from which the game-theoretical applications of my paper build upon. The goal of this analysis is simple; predict what the optimal distance of both the four-point line and extended three-point line would be. In order to achieve this, I use historical data and projection techniques, tailored toward the information I was able to acquire about each type of shot. To first frame the discussion, I will outline some important details about the expected value of shots taken in the NBA.

5.1 Expected Value Considerations

In order to understand the dynamics of a game with the complexity of basketball, a good starting point is to consider the expected value of each scoring opportunity. For the purposes of this research, I will omit free-throw shooting and look only at two and three-pointers. In the current NBA (2023-2024 season), players are shooting two-point shots at a rate of 54.9% and three-point shots at 36.3%. The expected value of taking these shots is calculated very simply by:

$$\begin{aligned} \mathbb{E}\left[\text{Points}_{2\text{pt}}\right] &= (2\text{ptFG}\%) \times 2, \\ \mathbb{E}\left[\text{Points}_{3\text{pt}}\right] &= (3\text{ptFG}\%) \times 3. \end{aligned} \tag{6}$$

For the 2023-2024 season, the observed expected values are 1.098 for three-point attempts and 1.0904 for two-point attempts. I extrapolated this calculation over time to create the following plot which displays expected value over time since the inception of the three-point line.

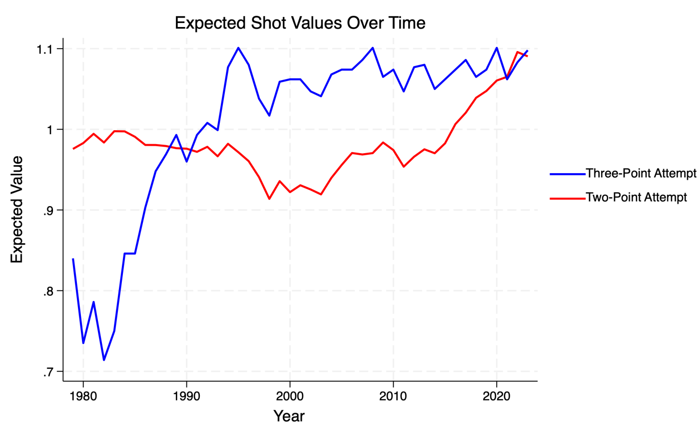

Figure 11: Expected Shot Values Over Time (Three & Two)

Notably, from around 1990 until 2020 there was a massive unexploited gap between these expected values; the three was significantly more valuable than the two. At the end of Subsection 4.2, I referenced a "clear game theoretical explanation" for the clear, positive relationship between three-point attempts and wins found in Table 9. This gap in expected shot value is certainly a major contributor; three-point shots were simply underutilized during this time period.

Although the increase in expected shot value for three-pointers from 1980 to 2024 is likely due to an increase in skill (players began practicing shots from that distance because before they were not very practical), this has leveled out over approximately the last decade. What hasn't leveled out, however, is the expected value of the two-point shot. The same logic does not follow here; one cannot simply assume that the only factor contributing to this increase is players getting better at shooting twos (due to the fact that the two-point shot has not changed for almost a century). What needs to be considered, instead, is the frequency at which these are being attempted. The following figure shows the relative frequencies since the inception of the three-point line:

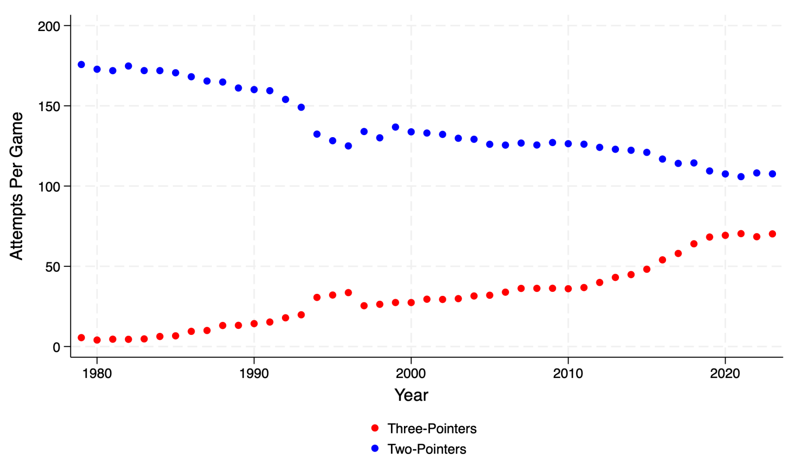

Figure 12: Scatter Plot of Two-Point & Three-Point Attempts Per Game

Unmistakably, the two have converged towards one another over time. This convergence is quite smooth, aside from a three year deviation from the 1996-97 season, where the three-point line was moved forward. Connecting this to the expected value discussion, it is clear that with the reduction of two-point attempts, the quality of these attempts has increased, thus increasing their expected value. In basketball terms, the NBA has likely seen a significant decrease in the "mid-range" game, which, as depicted in Table 4, is significantly less efficient than shooting close to the rim. This idea has been confirmed in many papers including Shawn Kilcoyne's 2020 thesis which talks extensively about the decline of the mid-range shot in modern basketball (Kilcoyne, 2020). Teams have begun to understand that in order for two-point shots to be worth shooting, these relatively inefficient shots need to be eliminated from the majority of players' strategies.

While the expected value of two and three-point shots have converged to nearly the same level, the same complaints about the volume of three-point shooting will likely persist until a rule change is implemented. The two proposals have two different arguments as to how to combat this issue, but hopefully both will cause the frequency of three-point shooting to decrease over time.

5.2 The Four-Point Line

In this section, I analyze the league-level data with a specific focus on understanding how I would implement the four-point line. To begin the process, I will determine the optimal distance for the line; this will shape the way the game evolves. Although no direct four-point shooting data in the NBA exists, I made the baseline assumption that the skill of the current NBA player from four-point range can be extrapolated from current data shooting three-point shots (from further distances). As illustrated by Figure 7, this relationship is fairly linear and visible. The goal of this rule change is to create this hypothetical four-point line at a place where shooting a four-point shot has approximately the same expected value as shooting a three-point shot, such that there is no dominant strategy. In the following figure, I graphed the two and three point expected values for 2023 in the same plot as a hypothetical four-point expected value situation. This used the same data used to create

Figure 7 and calculated the theoretical expected value for each potential distance:

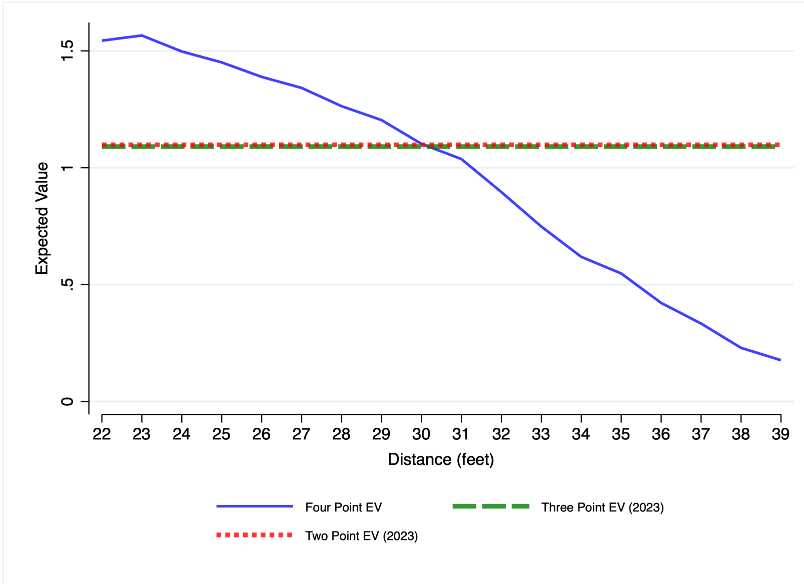

Figure 13: Expected Value vs. Distance Graph

Note that the two and three-point expected values are almost identical as expected (Figure 11) and that they intersect with the hypothetical four-point expected value at approximately 30 feet. This point of intersection makes complete sense; the NBA's observed field goal percentage on 30 foot shots is 27.56%, or in expected value terms, 1.102 (very close to our 2pt/3pt EVs). However, this is not the only thing to take into consideration when placing the four-point line. There are two issues with a 30 foot 4-point line; it does not account for any possible externalities of adding a four-point line, and also does not take into account the four-point skill development of players over time.

5.2.1 Possible Externalities

In order to address the former of these issues, I want to look back at the impact of the addition of the three-point line in 1979. It may be the case that by adding an incentive to shoot and defend further on the perimeter, the two-point efficiency of players increased. Figure 4 displays how two-point percentage changes over time, but this section will provide a more in-depth analysis using an Interrupted Time Series Analysis (ITSA).

An ITSA evaluates whether or not an interruption (event) has an effect on an outcome variable. In this case, the interruption event is the addition of the three-point line in 1979, and I want to test whether or not this has an impact on future two-point percentages. In this analysis, I used data from the inception of the NBA in 1949 up until 2023. I specified an indicator variable for post-1979 seasons, and an interaction term between that and the linear time trend centered at 1979. I used Newey-West standard errors with a lag of one in this analysis, which accounts for potential heteroskedasticity and autocorrelation. The regression looks like the following:

$$TwoPT\%_t = \beta_0 + \beta_1(Time_t) + \beta_2(Post1979_t) + \beta_3(Time_t \times Post1979_t) + \epsilon_{i,t} \tag{7}$$

Table 12: ITSA Model for Two-Point Percentages Over Time

Variables	**Coefficients**
time	**0.00514*** **
	(0.000547)
post1979	-0.0216**
	(0.0107)
timex1979	-0.00448***
	(0.000640)
Constant	0.498***
	(0.00811)
Observations	78

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

This table suggests a general positive trend over time, and a slight negative impact on two-point efficiency from the post1979 binary variable and the interaction term. Importantly, due to the small magnitude of the coefficients, it is possible that these results are reflective of a different issue despite their statistical significance. Immeasurable small changes in the way games are officiated, for example, could have been part of the reason that the coefficients are negative. The key takeaway here is that over time, two-point percentage trends upward, and it is possible that the implementation of the three-point shot had a small, negative impact on two-point efficiency. As for the four-point mode, this is good news; I do not want the four-point shot to create even more incentive to shoot threes.

5.2.2 Nonlinear Fit Model

As discussed previously, there is still one major issue to address; four-point shooting skills will develop disproportionately over time after this rule change. In order to address this, I sought a way to fit a nonlinear model to the relationship depicted in Figure 5, and thus model the development of four-point skills in a manner very similar to how three-point skill developed. Once I obtain this model, I will then apply it to a target efficiency for four-point shooting. When inspecting this relationship to determine what type of functional form would most accurately reflect it, I landed upon a logistic (S-Shaped) curve, which looks like the following in functional form:

$$3pt\% = a + \frac{b}{1 + e^{-c(season-d)}} \tag{8}$$

The components of this form include: a (the minimum / horizontal asymptote), b (the height: $a+b$ is the maximum), c (the logistic growth rate or steepness), and d (the value of the curve's midpoint). With this function, I can run a Nonlinear Least Squares Estimation using the historical three-point data to obtain a parameterization of this curve. After doing so, I obtained the following values for the parameters:

Table 13: Parameter Estimates

Parameter	Value
a	0.2470682
b	0.1092942
c	0.3904473
d	1986.738

Now that I have these values, I can graph the fitted equation alongside Figure 5. Below are the results:

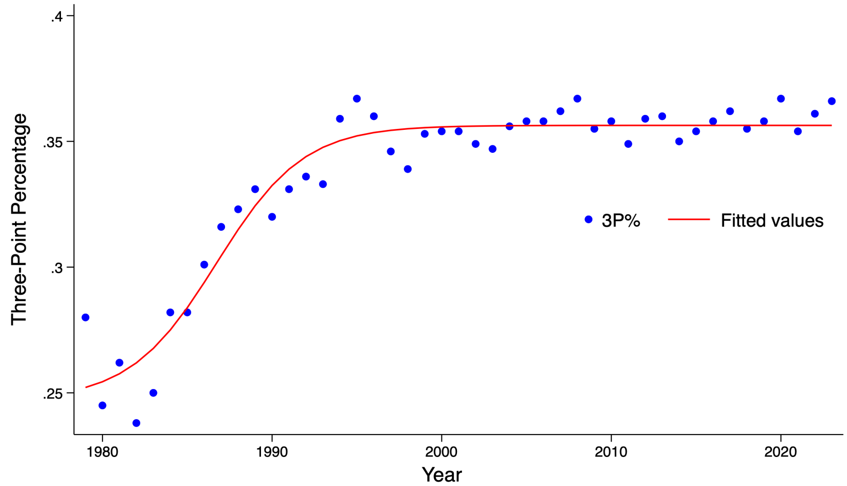

Figure 14: Logistic Fit of Three-Point Percentage Over Time

In general, this seems like a good estimate of the observed relationship between three-point percentage and time. Not only is this useful for modeling the past and present, but it can also help predict the future. Based on this fitted model, three-point percentage will level at approximately 35.636%, or an expected value of (.35636*3) = 1.069 over the next 20 years. To apply this to the four-point line, I want to target a plateau four-point percentage of (1.069/4) = 0.26725 or 26.725%. Using this model, I now have to generate the parameters for the four-point estimation. The target plateau entails that $a + b = 0.26725$ (the maximum). The height of change (b) should mirror that of the previous estimation, adjusted for magnitude. The steepness of the curve and midpoint will remain constant (so c and d will remain the same. With all of this information in mind:

$$b = 0.1092942 \times \left(\frac{0.26725}{0.35636}\right) = 0.0819645$$

$$a = (0.26725 - b) = 0.1852854$$

$$c = 0.3904473$$

$$d = 1986.738$$

Parameter	Value
a	0.1852854
b	0.0819645
c	0.3904473
d	1986.738

Table 14: Parameter Estimates

With these parameters defined, I can now graph the projection of four-point percentage over time. One thing to note when looking at this figure is that I adjusted the x axis to begin in the present day. This is intended to reflect the efficiency development if the line were added right now. The projection is shown below:

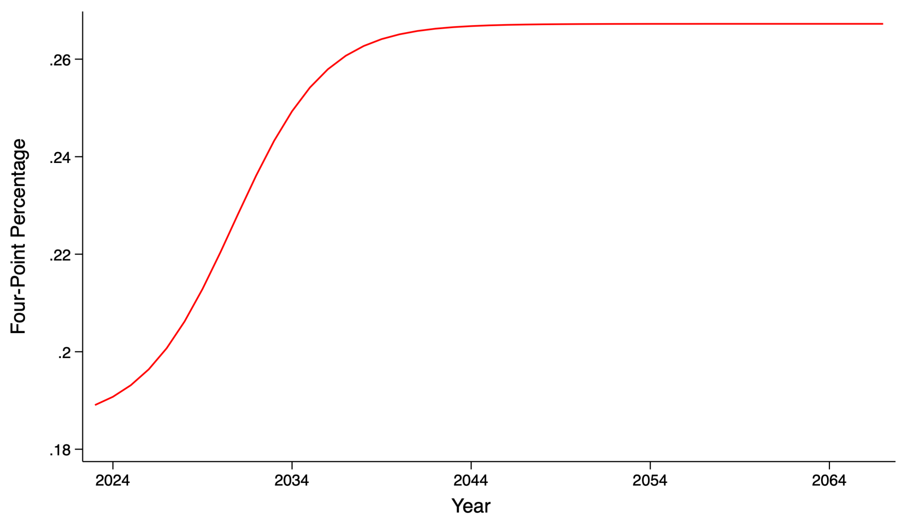

Figure 15: Projection of Four-Point Percentage Over Time

This is exactly the anticipated result; it exhibits the same properties as Figure 14, with a lower intercept, lower horizontal asymptote, and smaller magnitude. Not only does running the NLLS estimation provide this figure, but it also provides a projected percentage value for each year. In the projection, the 2024 value is 18.91%, meaning that the model suggests this as the "opening" percentage in order to follow this growth trend. Looking back at the data I used to make Figure 7, the implied distance for the four-point line is 33 feet. With an expected value of 0.7564, the four-point shot would clearly not initially be very valuable in most circumstances. However, this is similar to the three-point line development; only the very best shooters would use it at the start, and over time more players will develop the requisite skill for it to become useful. The end-of-game excitement factor of adding this line would still be very relevant. Below is a diagram depicting the dimensions of an NBA court and what this four-point line would look like (not to scale):

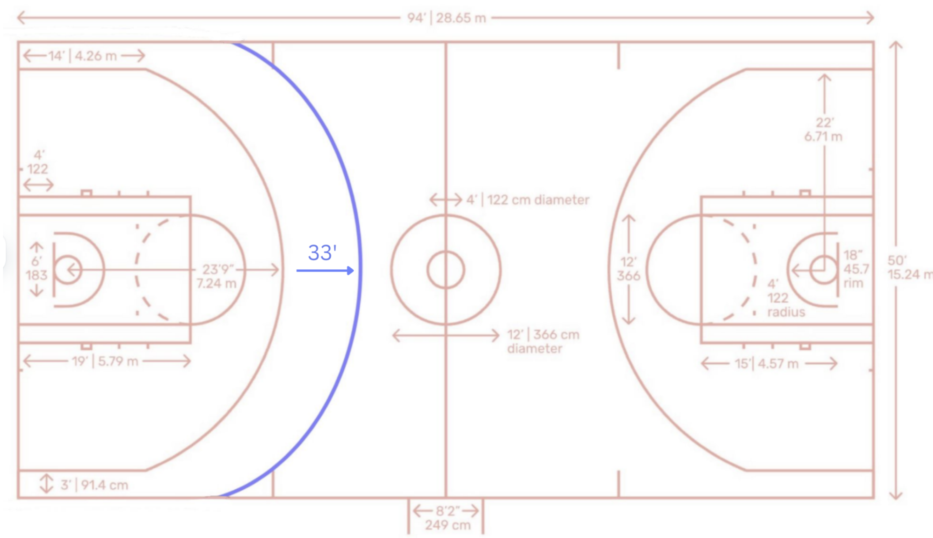

Figure 16: NBA Court Dimensions With a Four-Point Line at 33 Feet

The ramifications of this line are impossible to perfectly predict. However, it is near certain that the addition of the 33 foot four-point line would increase the diversification of shots, change offensive and defensive strategies, and significantly alter the way end-of-game basketball is played. Whether or not these are positive or negative externalities is arguable, but the immense impact on the league is irrefutable. Later in the paper, I delve into more specifics about the game-theoretical implications of the addition of the four-point line.

5.3 Extended Three-Point Line

As previously mentioned, while the four-point line does counter the shot diversification problem, it is possible that the true issue lies in the monotonous nature of extensive distance shooting. To counter this problem, one of the most commonly discussed solutions is moving the current three-point line backward. This section will seek to analyze the most efficient way to do so under the conditions of the modern NBA.

Currently, the NBA shoots threes at a higher frequency than ever before, therefore I am aiming to shift this frequency distribution toward players shooting less of these shots, and in turn more two-pointers. In order to understand the target distribution, consider the following; since the turn of the twenty-first century (approximately the amount of time three-point percentages have plateaued, see Figure 5), the peak regular season NBA ratings came during the 2011-2012 season (Sports Media Watch, 2025). For the purposes of this analysis, I will use the three-point shot frequency distribution from this time period as a target. Below are the distributions, calculated at a player level and binned into histograms for the 2023-24 and 2011-12 seasons respectively:

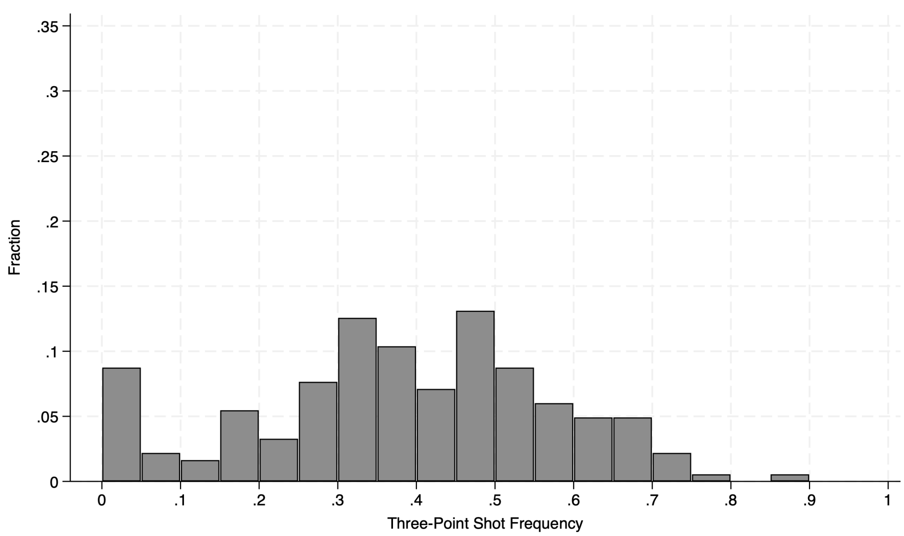

Figure 17: Three-Point Shot Frequency Distribution(Independent Players) 2023-2024

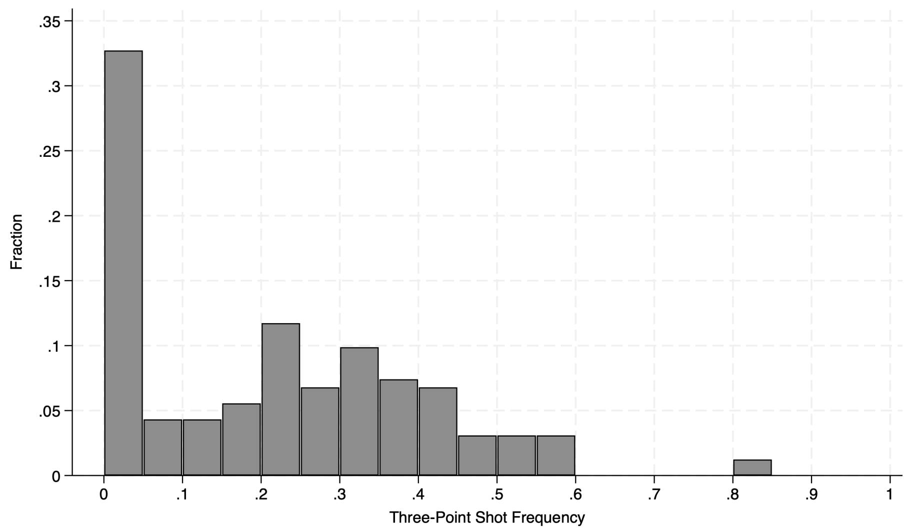

Figure 18: Three-Point Shot Frequency Distribution (Independent Players) 2011-2012

Noticeably, the target distribution is skewed significantly leftward from the current one, with the majority of players shooting threes at a frequency less than 30% of their total shots. With this information in mind, along with our statistical understanding of current NBA player skill, I now attempt to determine the line length at which the distribution will look most similar to the 2011-2012 distribution.

To achieve this goal I utilized the Kolmogorov–Smirnov (K-S) Test (Jain, 2024). This statistical method allows us to compare two samples, understanding whether or not they follow the same probability distribution, and if not, how similar the shape of the distributions are. For the sake of this paper, I ran the K-S test on seven hypothetical three-point line distances (24ft, 25ft, 26ft, 27ft, 28ft, 29ft, 30ft), along with the current NBA shot distribution. Each shot taken below the hypothetical distance, regardless of if it had previously been counted as a three-point shot, was counted as a two (obviously not including the K-S test using current NBA distances). Thus, each foot of distance back lended less and less hypothetically attempted three-point shots, and the K-S test was utilized to analyze which of the distributions was most similar to the 2011-2012 distribution. The K-S statistic ranges from 0-1, where 0 represents an identical distribution and 1 represents two totally different distributions. The following are the results:

Table 15: Kolmogorov-Smirnov (K-S) Test Results

Distance	K-S Statistic
Current NBA (22-23.75 feet)	0.4075
24 feet	0.3372
25 feet	0.2561
26 feet	**0.2084**
27 feet	0.2694
28 feet	0.4429
29 feet	0.6220
30 feet	0.6483

Although all of the comparisons displayed variation relative to the target distribution, the 26ft distance yielded the smallest K-S statistic, and was thus most similar to the 2011-2012 season's shot frequency distribution. Below is the distribution with the hypothetical 26ft three-point line in place:

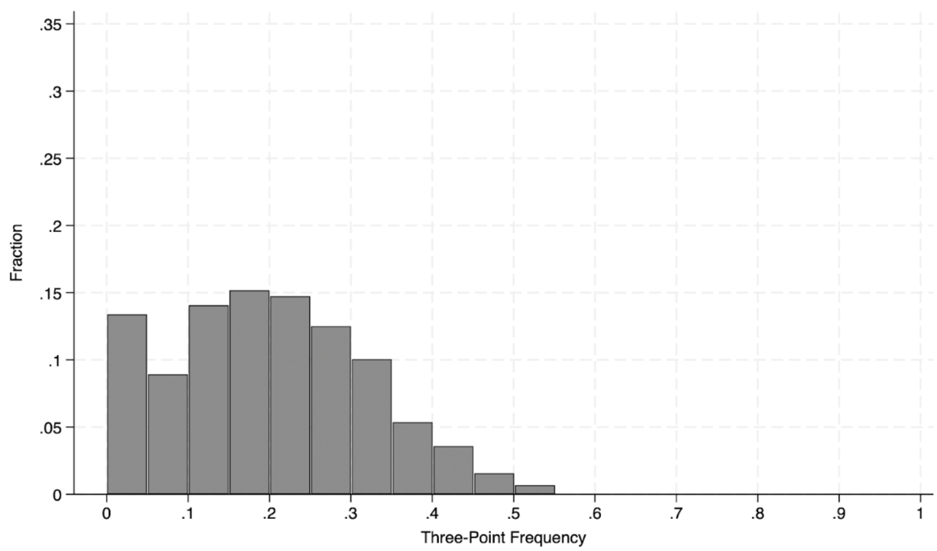

Figure 19: Hypothetical Three-Point Shot Frequency Distribution (26 Foot Line) 2023-24

This distribution, while not entirely identical, bears much resemblance to Figure 18. The 26 foot three-point line will incentivize teams to shoot more twos, and more specifically have only the most skilled shooters on their team take three-point shots. Gameplay, at least in terms of the balance of shot selection, will likely more closely resemble 2011-12. Below is an interpretation of what the NBA court would look like with this change (not to scale).

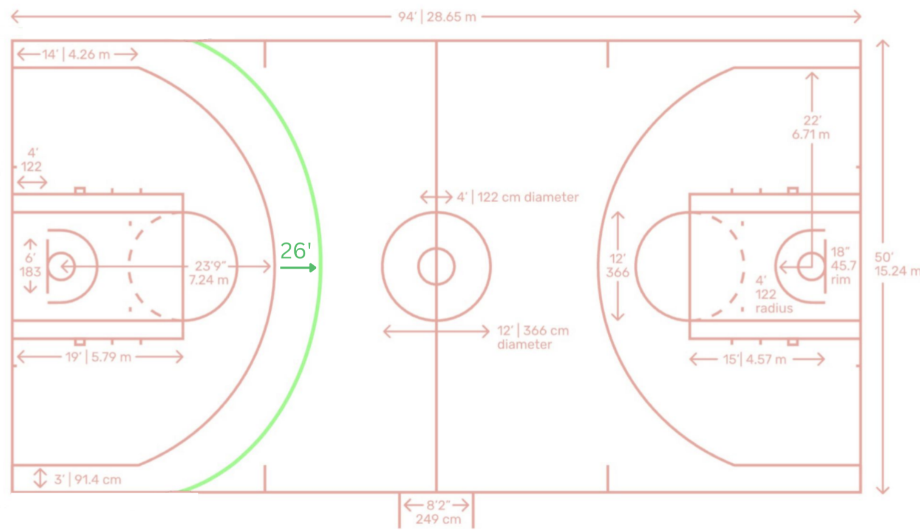

Figure 20: NBA Court Dimensions With a Three-Point Line at 26 Feet

Now that I have asserted the length of the extended three-point line to be 26 feet, this coupled with the 33 foot four-point line will be used in the next section to understand game-theoretical implications of these potential changes.

6 Game-Theoretical Implications

This section will use game-theoretical methods to determine the potential implications of these proposals in the future. I will do so in a way that allows us to compare the current state of the league with a new, hypothetical one. Before I dive into specifics, I also wanted to note that I used Fichman & O'Brien paper as an inspiration for my methodology throughout this section (Fichman and O'Brien, 2019).

6.1 The Model

Using the zones defined in Figure 10, I normalized shot data points from NBAStuffer to one side by rotating them about the midpoint of the court. With these normalized data points from the 2023-2024 season, I then categorized each shot into a zone (from 1-11). For simplicity, observations which were on the border between two zones were dropped (influencing under 1% of the data points). Table 4 provides summary statistics for the resulting zone categorizations.

In order to begin the analysis, I need to define a few important characteristics. The summation (from zone 1 to 11) of the product of shot frequency, ω, efficiency α, and point value of the shot, κ, together represent the expected payoff, μ:

$$\mu = \sum_{i=1}^{11} \omega_i \alpha_i \kappa_i \tag{9}$$

I used the aggregate shot data from 2023-24 discussed in Section 3.5 to obtain the values of these variables across the entire league. For every team and zone, I calculated both their offensive and defensive expected payoffs, along with the standard deviation and variance of these metrics. With these calculations, I can define the offensive and defensive Sharpe ratios as follows:

$$\text{Offensive} = \left(\frac{\mu_{\text{Offense}}}{\sigma_{\text{Offense}}}\right) \quad \text{and} \quad \text{Defensive} = \left(\frac{\mu_{\text{Defense}}}{\sigma_{\text{Defense}}}\right)$$

Using these Sharpe ratios, I then set up a constraint maximization which represents the Stackleberg leader's problem as defined by Fichman & O'Brien:

$$\textbf{Maximize } w.r.t.\ \boldsymbol{\omega} \quad \ln\left(\frac{\mu_{\text{Offense}}}{\sigma_{\text{Offense}}}\right) - \ln\left(\frac{\mu_{\text{Defense}}}{\sigma_{\text{Defense}}}\right) \tag{10}$$

Subject to:

$$\sum_i \omega_i = 1 \tag{11}$$

$$\omega_i \geq 0 \tag{12}$$

Before diving into the results of this optimization problem, there are a few necessary clarifications I need to make. While I did run this optimization for every team in the NBA, I did not do so for every combination of teams. That is, the defensive values inputted for the payoff and standard deviations were derived from the league average values. Thus, every team's offensive output was maximized subject to playing against the average NBA defense, rather than maximizing for every single matchup. Additionally, in order to avoid corner solutions, there needed to be some type of penalty imposed for shooting excessive amounts of shots from one zone. This holds intuitively; if a team decides that they are most efficient only shooting from the right corner (zone 5), the opposing team will react by defending it more effectively, thus lowering the efficiency of the offense. To do this, I decided to impose a one percent field goal percentage penalty toward every additional percent of a team's shots that came from a given zone. For example, the Atlanta Hawks shot 38.5% from the right corner (zone 5) while taking 7.66% of their shot attempts from this zone. In my model, the next additional percent share of shot attempts (up to 8.66%) would be at 37.5% efficiency, and so on. This incentivizes a more balanced selection of shots while simultaneously allowing for extremely efficient shots to be rewarded. Note that while this is a somewhat arbitrary penalty, the true benefit of the analysis will be comparing what this maximization does when changed to account for a four-point line or an extended three-point line. Thus, all of these will account for this penalty and I will be able to extrapolate general conclusions regarding the impact of these rule changes on the NBA from this comparison (not as much the raw numbers, while they are valuable).

6.2 Results

To begin, I ran this optimization problem on the shot data from the current NBA, changing none of the zone definitions or boundaries. This used all of the shots from the 2023-2024 NBA season. When reading these results, it is important to note that zones 1, 2, 3, 6, 7 and 8 are worth two points, while zones 4, 5, 9, 10, and 11 are worth three. The results are below:

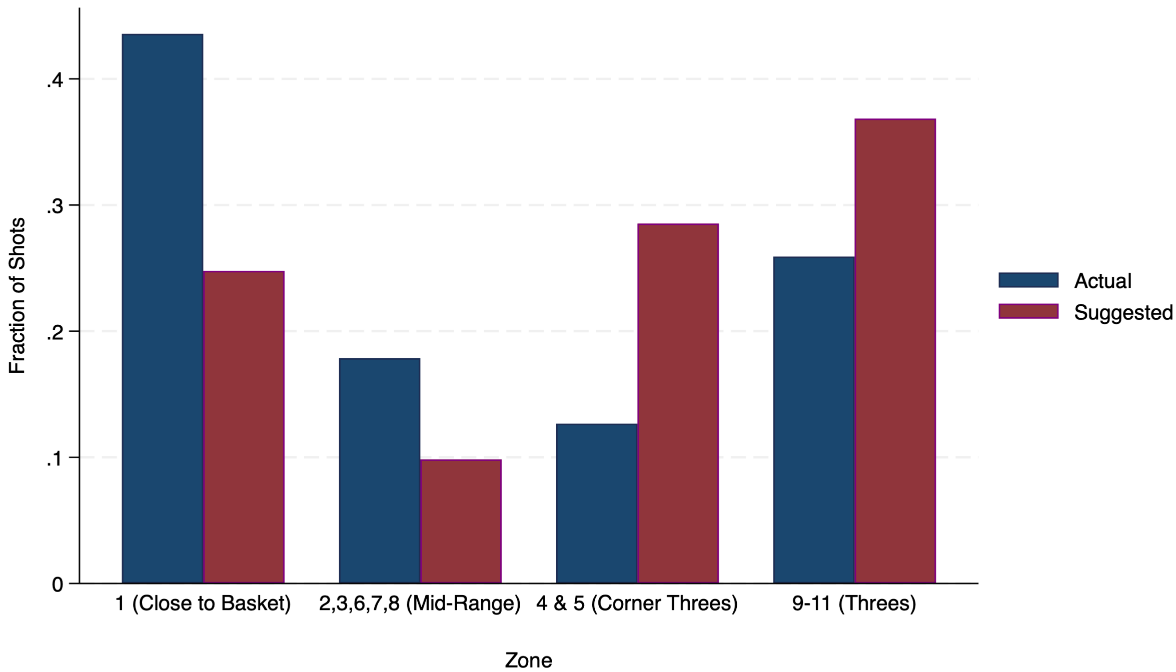

Figure 21: Maximization Results – Current Rules

The model suggests a decrease in shot frequency from all two-point zones, and subsequently an increase from the three-point zones. This indicates that, even with the aforementioned prevalence of current three-point shooting, shooting more threes is still optimal. While the exact percentage results from this figure should be interpreted cautiously (because of the 1% penalty), these results are still significant. This establishes a control to evaluate how the 26 foot three-point line and the 33 foot four-point line might impact how the game is played.

To continue, I modified the dataset significantly by adjusting the zones to reflect a 26 foot three point line. Most importantly, in this hypothetical, corner threes are now worth two, so zones 4 and 5 will be bunched in with the "Mid-Range" shots. On top of that, threes from zones 9, 10, and 11 are pushed back, meaning that the ones who were between 23-26 feet are now apart of either zone 6,7, or 8. Below is a graphic representing the new zones (compare to Figure 10):

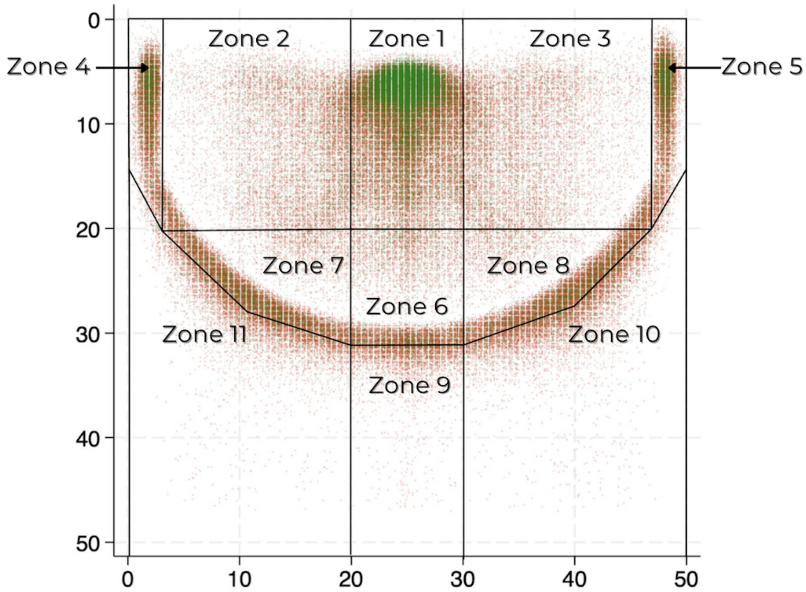

Figure 22: Shot Chart Zone With Definitons (26ft 3pt Line)

Using these zone definitions, I then ran the maximization problem again with the same constraints and penalty. The results are shown here:

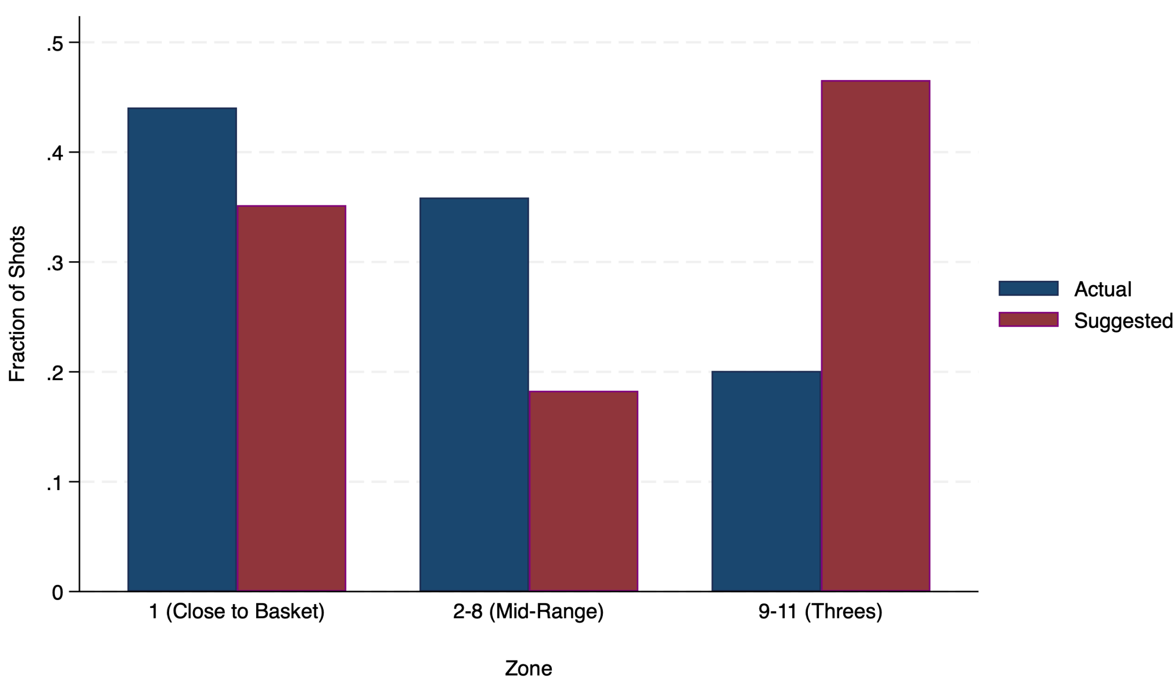

Figure 23: Maximization Results – 26ft Three-Point Line

At first glance, it may seem like these results are not promising due to the stark increase in threes shot from zones 9,10, and 11. However, when accounting for zones 4 and 5, it becomes clear that in the aggregate, this situation causes the model to suggest significantly less three-point shooting(47% compared to 65% before). On top of this, I would argue that these results are even more promising considering the fact that the sample efficiency used to calculate expected payoff from beyond 26 feet is more often shots taken by some of the best shooters in the league. That is, it is likely that there are a higher proportion of shots taken in these zones by players like Stephen Curry, with significantly higher shooting ability than league average. This would lead to an overestimate in the amount of threes suggested. In general, these results are promising and suggest that moving the three-point line backward would indeed have a massive impact on the amount of threes taken.

Next, I will analyze the four-point scenario. In this case, all of the zones remain the same as in Figure 10, and zone 12 is added as a four-point line arch at 33 feet. Below are the maximization results:

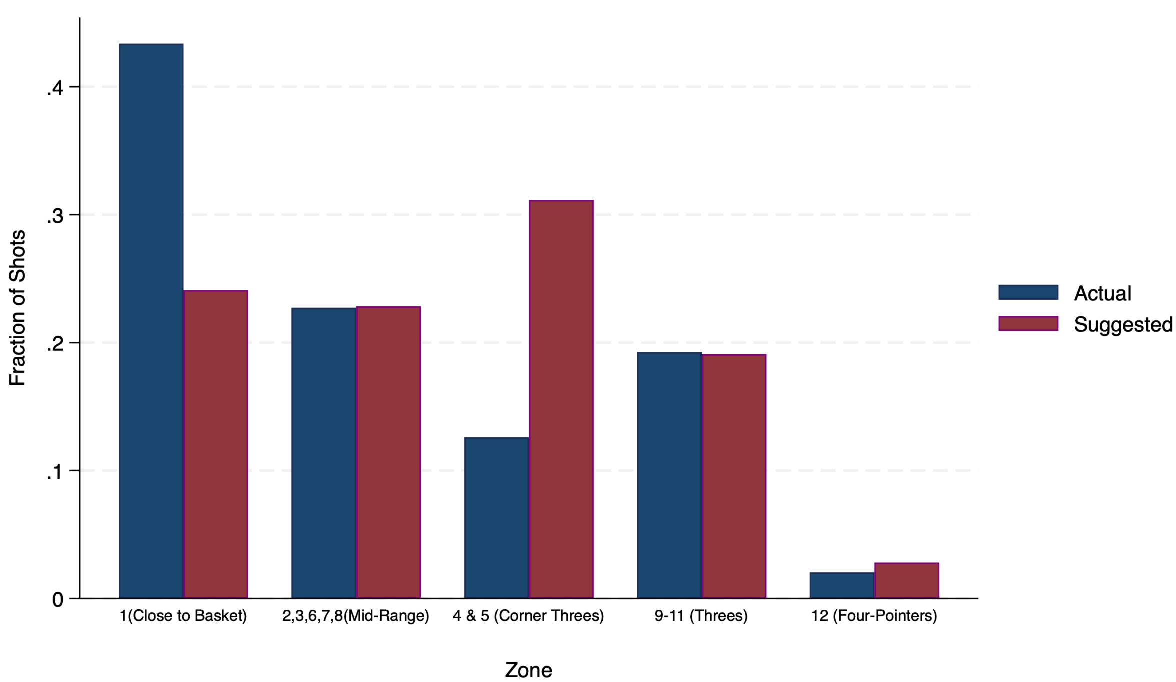

Figure 24: Maximization Results – 33ft Four-Point Line

Interestingly, this change had a significant negative impact on the frequency of threes suggested from zones 9-11 being taken, alongside a slight increase in the amount being taken from the corner. Due to the fact that this was calibrated with an observed four-point percentage of only 18.91%, the model does not suggest many four-pointers being taken at this moment in time. It is important to note, however, that if the model was run with a zone 12 (from 33 ft out) before it was deemed worth four points, it would in all likelihood suggest 0% of shots to be taken from that zone (EV: 0.1891 * 3 = 0.5673).

After I looked at these results, I concluded that both moving the three-point line back and adding a four-point line do have some level of positive impact (the magnitude of which will be discussed more later in the paper) and consequently decided to run the optimization with both solutions combined. This entails that the zones were defined as in Figure 22 but with a four-point arch added at 33 feet, zone 12. Below are the results:

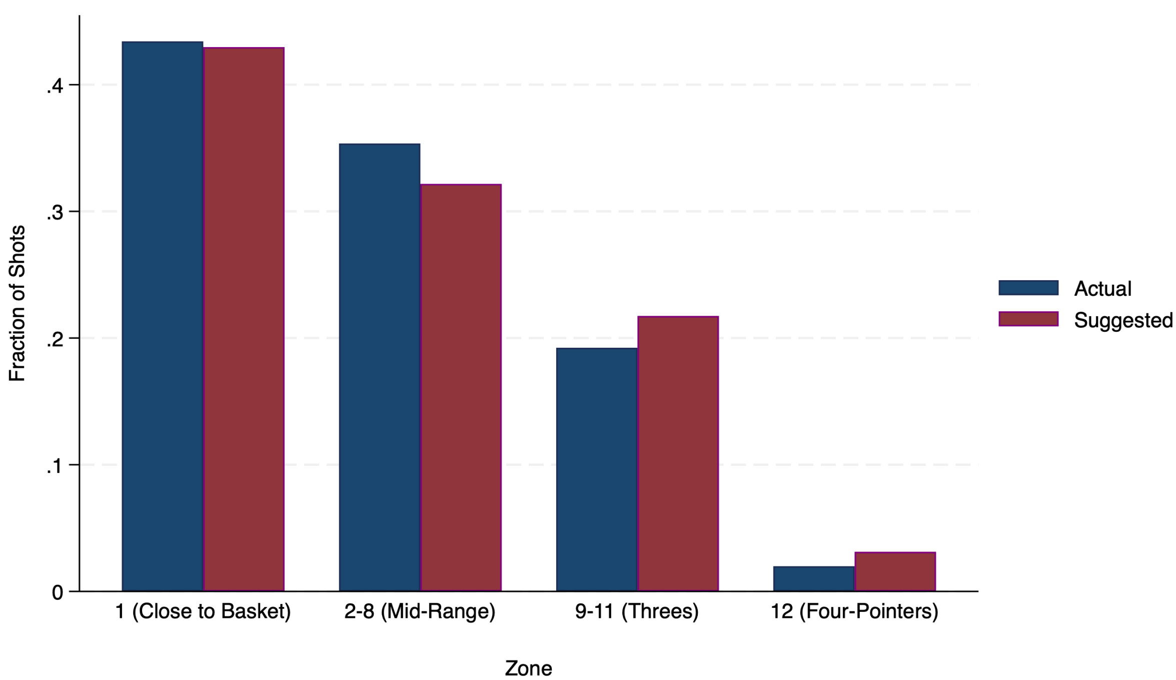

Figure 25: Maximization Results – Both Lines Implemented

Combining both of these rule changes had a more substantial impact than either one individually.

While the model does still suggest to shoot more threes than currently observed from zones 9, 10, and 11, it is a significantly smaller total amount than before (because zones 4 and 5 are no longer threes). This result is extraordinarily promising; suggested long distance shooting totals only 25% (22% three-pointers and 3% four-pointers). Below is a table summarizing the results from all four of these optimizations:

Scenario	**Current Rules**	**26 ft 3pt**	**33 ft 4pt**	**Both Lines**
Paint (zone 1)	25%	35%	24%	43%
Midrange (zone 2,3,4,5,6,7,8)	10%	18%	24%	32%
Corner 3s (zone 4,5)	28%	–	31%	–
Other 3s (zones 9–11)	37%	47%	19%	22%
4-pointers (zone 12)	–	–	2%	3%

Table 16: Optimal Shot Distributions Under Hypothetical Rule Changes

An important note is that as time progresses, players will continue to increase skill in all facets of their game, but likely specifically in deeper shooting as was the case with the advent of the current three-point line. Therefore, as depicted in Figure 15, four-point efficiency will likely increase over time, causing the frequency of these shots taken to increase with it. Additionally, while these suggestions are what my model deems as "optimal", these frequencies are, in all likelihood, reflective of teams' future shooting decisions. While my model suggests teams right now should be shooting over half of their shots from the three-point line, the observed frequency is only approximately 42% (but trending upward). Teams are not perfectly optimizing, but as time progresses and analytics improve, they will continue to make decisions closer to this optimality.

7 Discussion

In this section, I provide a more in-depth interpretation of the most important results from this paper. While all of the figures and tables I reference are from earlier sections, this section will emphasize their significance and their connection to one another. Then, I discuss the implications of these results for the NBA, and what I would recommend as a rule change. Because basketball is such a dynamic game, I would be remiss to not mention the limitations of this paper. It is impossible to perfectly predict how offensive and defensive strategy would react to these changes, and thus I need a limitations section to discuss these issues and their implications. I also discuss opportunities for research with regard to these limitations, other ways of analyzing these ideas, and other potential rule change ideas.

7.1 Interpretation of Results

Section 4, Section 5, and Section 6 all contain unique and intriguing results, together constructing this paper's argument. In the following subsections, I discuss these each individually, parsing the important interpretations of these results.

7.1.1 Motivation Results

Section 4.1 established a mathematical relationship between increased three-point shooting and increased variance of point differential. I then tested the variances of two groups of years, one group where 3.5 threes were attempted per game, and the other with 32.4 attempts per game (approximately equal PPG). The Levene test for equivalence of variances in Table 8 resulted in a p-value of 0.00000033. Any p-value below 0.05 labels the two variances as statistically significantly different, which is the case here. The fact that this p-value is so low speaks to the clear difference between the variances of the two groups. The W50 and W10 alternative location estimators also support this hypothesis. A distribution with a higher variance necessarily entails a larger number of results significantly higher than the mean, given equal sample size. In basketball terms, more games will be decided by more than 20 points, for example. Table 6 accentuates this effect with a positive, statistically significant coefficient of 0.957 (on the Group 2 binary variable). This shows that not only does the variance of point different increase, but the average absolute value of it does as well. The positive movement away from zero (which would theoretically be the most competitive and entertaining mean point differential) necessarily entails that these games on average are not as close. These two effects are synergistic; they are together amplifying the effect on point differential. This effect (the increase of mean and variance of point differential) is a negative externality of higher frequency three-point shooting, motivating my research.

While I already established empirically that teams are shooting more threes, I wanted to econometrically depict the incentives which arise from doing so. Controlling for salary cap spending and year, Table 9 shows with a statistically significant result that three-point attempts yield more wins. Importantly, this is testing on attempts, not makes or efficiency. This is likely a result of the discrepancy in expected value of three-point shooting displayed in Figure 11. To create the dataset used Table 10, I adjusted for the inflation in salaries over an eight year span. While this does not directly co-move with the NBA salary cap, the time fixed effects account for this discrepancy. After additionally setting controls for both position and year, there exists a statistically significant (positive) relationship between three-point shooting rate and salary. Again, this is importantly not efficiency or makes, it is simply fraction of shots coming from three. Together, these regressions were intended to display the motivations on both a team and player level to shoot more three-pointers. Similar to the point differential discussion, these both amplify the effect seen on three-point shooting; they are both incentivizing the continuation of the positive trend over time.

Through this motivation section, I aimed to display a directly quantifiable negative impact of three-point shooting, and then show how teams (and players) will keep doing it because they are incentivized to do so. Whether you agree or disagree with my claim that lack of shot diversification is bad for the viewing experience, the impact on point differential is certainly a negative ramification of shooting more threes.

7.1.2 Analysis Results

In this section, I defined the distances of both the four-point line and the extended three-point line. I started by defining and visualizing the expected value of shots in the

NBA over time. From this, I extrapolated that the goal four-point percentage should be approximately 27.6% to obtain an equal absolute value to the two and three point shots. The ITSA in Table 12 defines the “time series interruption” as the implementation of the three-point line, and analyzes its impact on two-point efficiency. The goal here was to determine whether or not I had to adjust for any type of predictable positive impact on three-point shooting efficiency when adding a four-point line, due to increased spacing or otherwise. However, these results suggest quite the opposite. Significant at the 95% confidence level, the “post1979” indicator suggests that this implementation caused a slight, negative impact on two-point shooting. Because the point of this paper is to decrease the volume of three-point shooting, if this externality were to occur in my four-point line suggestion, it would not be a bad thing. It is important to note here that while this is my best to model any potential externalities, because this is a new concept, I cannot be certain that what happened with the introduction of the three-point line would happen with the four-point line. I did not adjust for any externality in my analysis, and continued forward. Using a NLLS estimation, I fitted a curve to the development of the three-point shot over time in Figure 14. The resulting curve was extremely accurate and seemingly reflected the relationship quite well. The pseudo R-squared value of this correlation is 0.9287. Before I applied this to the four-point line, I adjusted for the magnitude because it is more likely players are getting approximately 40% better than where they started at each type of shot, rather than this reflecting an absolute percentage. After scaling for this, I then obtained a curve in Figure 15 which represents my prediction for how the four-point line would develop over time. This resulted in a prediction of 33 feet being the optimal distance for the four-point line in order for it to develop to the 27.6% efficiency in approximately 20 years. This result is promising, as currently less than 3% of shots come from this range, indicating that incentivizing shooting from this range would cause some change in the shot distribution. The applications of this result will be further realized in the game-theoretical results section.

In the extended three-point line part, I compared several three-point shot frequency distributions using the Kolmogorov-Smirnov (K-S) Test. The goal was to craft a shot frequency distribution most similar to the 2011-12 season, as ratings peaked during this time (and three-point attempts per game were only approximately half of that of a game today). The results in Table 15 showed that the lowest K-S statistic (D) value was with a 26 foot three-point line at a value of 0.2084, indicating that this distribution was most similar to the target distribution. Note that while this is the most similar, the magnitude of similarity is tough to decipher. However, given these two are made from different datasets of shots (2023-24 season vs. 2011-12) a completely uniform match would be impossible, thus taking the smallest D determined the length of my line suggestion. Figure 19 presents the new distribution with a 26 foot three-point line. Comparing this to the current NBA in Figure 17, the difference is immediately visible, and the distribution is much more similar to the target. Now, I analyze how this 26 foot three-point line and the 33 foot four-point line would impact the game through a game-theoretical lens.

7.1.3 Game-Theoretical Results

I have established that Section 4 displays the negative repercussions and existing incentives for shooting threes, and that Section 5 sets the new lines at 26 and 33 feet respectively. In this section, I modeled how these changes would impact teams' shot making decisions. To do so, I used the maximization problem laid out in equations 9 through 12. Figure 21 depicts some of the most crucial information in this paper. This shows how the model suggests current NBA shot selection should look, with 65% of shots coming from three, 25% from the paint, and 10% from the mid-range. The crucial aspect of this is that even with the current selection being heavily weighted in three-pointers, and a penalty for shooting additional shots from each zone, there is still incentive to shoot more threes. Figure 23 implements the 26 foot three-point line extension. The relative increases shown are the key here; more shots are suggested in zone 1 (25% to 35%), and the mid-range (10% to 18%), and less from three (65% to 47%). This logically makes sense, the expected payoff from three is significantly decreasing with a decrease in aggregate efficiency, α. Although zones 9-12 do increase in frequency, this can be attributed to some of the distribution of threes being taken away from the corners (which are no longer categorized as threes) and placed into these zones. These results are promising, and bring the suggested shot frequencies quite close to the current observed frequencies from each of these three shot categories (a significant improvement from the current suggestions).

The four-point line results are less straightforward. Shown in Figure 24, frequency from zone 1 decreases slightly (25% to 24%), the mid-range increases (10% to 24%), corner threes increase (28% to 31%), other threes decrease (37% to 19%), and shots from beyond 33 feet (now deemed worth four points) increase (1.5% to 2%). This result does make some logical sense, but is unexpected in the context of my model. This model does not directly factor inter-zone effects into the equation, thus this increase in mid-range shooting is surprising. However, I believe that this is because the model maximizes the Sharpe Ratio, and thus the lower variance of these shots ended up allowing the suggested frequency to remain similar to the observed frequency. Regardless, the outcome suggests a slight decline in three-point shooting and increases shot variation, both of which are positives in the overall outlook of this paper. With these unexpected results, I decided it would be interesting to combine the two rule changes and run the maximization problem again. Figure 25 displays the results with the implementation of both lines. In this, the fraction of shots suggested in zone 1 (25% to 43%), the mid-range (10% to 32%), and four-point range (1.5% to 3%) all increase. This combination of the two rule changes provides a result which decreases three-point shooting frequency (perhaps even too far) to just 22% of total shots. However, it is important to remember that four-point shooting will develop more efficiency over time in this model, and thus it is possible that once it reaches peak efficiency, distance shooting at a whole will return to its current frequency. The most impactful takeaways from these results are the following: moving the three-point line back to 26 feet directly achieves the desired result, and adding the four-point line produces intriguing results that are a bit more unclear.

7.2 Rule Change Suggestion

In the current NBA, I believe that three-point shooting is becoming an issue which will eventually need to be addressed. In this paper, I discussed the impact on the mean and variance of point differential, alongside some less quantitative rationale (decreased shot variation) in hopes to motivate this work. While my results suggest that both moving the three-point line back and adding a four-point line are plausible options, I believe moving the three-point line most directly solves the problem at hand. This will decrease the aggregate efficiency of the three-point shot, thus incentivizing teams to shoot more twos. While perfectly predicting the impact is not possible, I anticipate that teams' defense would focus more on protecting the paint, increasing the incentive to shoot from the mid-range as well (my model predicts this). The addition of a four-point line, while it may decrease three-point frequency, would potentially cause the same point differential issue once players begin to improve their shot making ability. Shown by the proof in Section 4.1, given equal frequencies, a four-point line would make this problem even worse. Because of this and the encouraging results from my analysis, I would suggest that the NBA move the three-point line back to 26 feet, as depicted in Figure 20.

7.3 Limitations

As I have stated numerous times throughout this paper, the dynamic nature of basketball cannot be understated. This makes perfectly modeling the sport impossible. Despite this, sports analytics is a growing field which provides useful insights helping teams perform closer to optimality. The truth is, however, that this optimality will never be reached and unpredictability in sports will always be a factor. In this paper, there were several points at which I had to make assumptions, or not adjust for certain things which limit the scale and applicability of my results. To begin with, as described in Section 6, I ran the maximization problem using a league average defense. This means that the model assumes teams would not adjust at all to their specific opponent, which is likely not an accurate assumption. Teams would likely shoot more in certain zones conditional on how well the opposing teams defend this zone, and so on. While this does mean that my results are not specifically accurate on a team-by-team basis, the fact that I used the league average defense means that my results are still applicable to a league-wide scale. Additionally, the 1% penalty introduced in this problem is quite arbitrary. Without it, many teams would simply shoot all of their shots from a few zones, but with it, it is hard to depict how accurate this penalty scale is (especially since it does not differ by zone). I combated this limitation by discussing my results mostly in terms of which direction the frequency shifted, rather than being too concerned about raw percentage shifts. This is certainly a part of the results where I am limited in my ability to prove their absolute accuracy (but the direction of shift still applies). Lastly, the choice of the 2011-2012 season used in Section 5 as the target distribution for the extension of the three-point line was also relatively arbitrary. There is no exact year that would be objectively better or worse for the NBA, and thus I used the year with the highest ratings. Clearly fan interest was high at the time, but I do not provide causal evidence to suggest that three-point frequency at this time was the reason people were so interested in the game. Generally speaking, I used this season to provide a realistic goal; a season not too far removed from the present but one where three-point shooting frequency was significantly lower than it is today. This is a limitation in the sense that one could argue a different time period would be better to model after, but not in the sense that it impacts the accuracy of my results. All three of these limitations are important to take into account when reading this paper, and allow room for this research to grow in the future.

7.4 Opportunities for Future Research

Utilizing economics and statistics to analyze sports is a place with unlimited future areas for research. The evolution of these sports, especially basketball over time cannot be understated. Related to my paper, it would be interesting to see how these results would change if someone implemented a dynamically adjusting defensive model to a similar maximization problem. Also, applying these potential rule changes and modeling how end of game situations would change is another path that would certainly extend the scope of this research. On top of these more directly connected topics, I think it would be quite interesting to look into two other rule change suggestions I have seen bouncing around; making dunks worth three, and removing the three-point line. The dunks worth three argument centers around the excitement of these plays and the fact that this would incentivize teams to attack and protect the basket more often than they currently do. Removing the three-point line would be an indication that players have gotten so good at these shots that it may be worth it to just do away with the additional incentive for shooting from further away. Both are interesting, and I would love to hear what types of takeaways people could gather from diving into these suggestions more comprehensively.

7.5 Conclusion

This paper examines the NBA's growing reliance on three-point shooting, considers potential rule changes, and analyzes how these rule changes could impact the game. I found statistically significant results which suggest that the mean and variance of point differential increase with three-point attempts. Additionally, I showed that salary (player level) and wins (team level) increase with three-point frequency. I then used statistical methods to determine the optimal distance of the extended three-point line and four-point line, and analyzed their theoretical impact on the game. This research motivates and curates an argument for changing the current NBA rules. Whether it be a 26 foot three-point line, a 33 foot four-point line, or another change, the league will undoubtedly need to continue evolving over time to account for the dynamic style of play. In a league where Anthony Edwards (one of the highest-flying players in the NBA), and Victor Wembanyama (a 7'5" center) are shooting more three-pointers per game than Steph Curry did in his 2014 MVP season, it might be time for a change.

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